EE4440 HW#6 Solution

April 13, 2011

1. Implement differential PCM in MATLAB, write a function that DPCM encodes a data stream, a function that DPCM decodes a data stream, and a script which tests the two.

   (a) Assume that you have some mechanism of letting the decoder know what initial condition is. Is this a realistic assumption? Why or why not?
   Solution: Yes, you as the designer could easily decide that this was needed. It simply requires that the first 8 bits transmitted be the starting condition, obviously there is a tradeoff between simplicity of implmentation and required accuracy of the solution.

   (b) Use the following data stream as a test: x=[127 125 123 121 119 120 121 122 123 124 123 122 121 120 123 126].

   (c) Plot your input signal, differentially encoded signal, and decoded signal in one figure using subplot and the plot command "stem."

   (d) Assuming that the given signal is an 8 bit signed signal, how many bits will you need to represent the encoded signal?
   Solution: Three bits will be needed, including a sign bit. That means we are saving more than half the bits required in this example.
   Solution to b and c:

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x=[127 125 123 121 119 120 121 122 123 124 123 122 121 120 123 126];

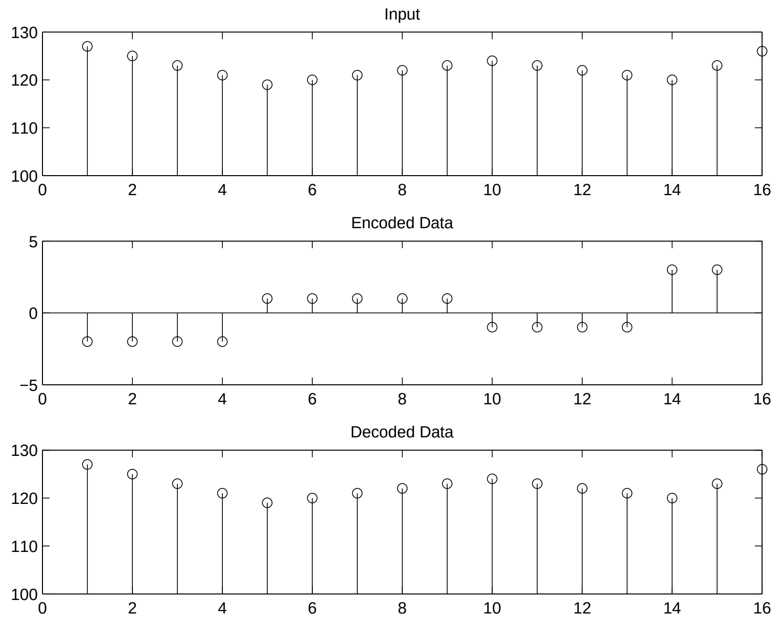
encode=diff(x);
decode=cumsum([127 encode]);

figure(1)
subplot(311);
stem(x)
axis([0 16 100 130]);
title('Input');

subplot(312);
stem(encode);
axis([0 16 -5 5]);
title('Encoded Data');

subplot(313);
stem(decode);
axis([0 16 100 130]);
title('Decoded Data');

print -deps hw6problem1figure.eps
```



2. What is the valid range of digital frequencies expressed as fractions of the sample rate? In radians per second?
Solution: Normalized or digital frequency takes on the values $\hat{f} \in [-0.5, 0.5]$as a fraction of $f_s$and $\hat{\omega} \in [-\pi, \pi]$ in radians. Often the fractions are given with $f_s$in the numerator, even if no sampling rate is assumed.

3. What are the maximum and minimum sampled frequencies expressed as a fraction of the sample rate? Given the following frequencies and sample rates find the digital frequency of the given frequencies at the given sample rate, and note what legitimate frequency will be interfered with in the case of frequencies that are above the Nyquist rate:

   (a) $f_s = 20,000Hz$ and $f = 39,000Hz$
   Solution: Note that $\frac{39}{20} = 1.95$, then subtract 2 to place in valid range, finding -0.05 is the valid digital frequency. This corresponding frequency given the sample rate is $20,000Hz(-.05) = -1kHz$ so the frequency that is interfered with is $\pm 1kHz$

   (b) $f_s = 40,000Hz$ and $f = 39,000Hz$
   Solution: Note that $\frac{39}{40} = .975$, then subtract 1 to place in valid range, finding -0.025 is the valid digital frequency. This corresponding frequency given the sample rate is $40,000Hz(-.025) = -1kHz$ so the frequency that is interfered with is again $\pm 1kHz$

   (c) $f_s = 80,000Hz$ and $f = 39,000Hz$
   Solution: The digital frequency is 0.4875 so there is no aliasing problem.

   (d) $f_s = 2000Hz$ and $f = 100Hz$
   Solution: The digital frequency is 0.05 so there is no aliasing problem.

4. What frequencies will alias to the following digital frequencies for the given sample rates?

   (a) $f_s = 20,000Hz$ and $f_d = f_s/4$
   Solution: (20,000)0.25=5000 so the set of frequencies is $\pm 5000Hz \pm 20,000n$ where $n \in ..., -2, -1, 1, 2, ...$

   (b) $f_s = 16,384Hz$ and $f = 3f_s/4$
   Solution: (16384)(-0.25)=4096 so the set of frequencies is $\pm 4096Hz \pm 16,000n$ where $n \in ..., -2, -1, 1, 2, ...$

   (c) $f_s = 20,000Hz$ and$f = 9f_s/16$
   Solution: $(20,000)\frac{7}{16} = 8750$ so the set of frequencies is $\pm 5000Hz \pm 20,000n$ where $n \in ..., -2, -1, 1, 2, ...$

(d) $f_s = 2000Hz$ and $f = -9f_s/16$
Solution: $(2000)\frac{-7}{16} = -875$ so the set of frequencies is $\pm 875Hz \pm 20,00n$ where $n \in ..., -2, -1, 1, 2, ...$

5. If you require 100db of SQNR how many bits do you need in your A/D? For 200dB? What is the SQNR for these common A/D bit widths: 8,12,16 and 24?
Solution: $SWNR = 6.02Q$ so for 100db you need 17 bits for 200dB you need 34 bits and for the given bit widths you get about 48db, 72dB, 96dB, and 144 dB.

6. Create a plot like Figure 6.2 in your book for a sine wave w/ frequency 100Hz. Sample the sinewave at the 5 times the Nyquist rate, sample your sinc shaped pulses at 50 times the Nyquist rate for the sinewave or more. Plot one complete cycle of the sinewave. Use the stem function to plot the sinewave samples, plot the sinc functions with dotted lines, and plot the sum of the sinc functions with a solid line. What is the error between the sum of your sinc functions and the sinewave?
Solution:

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T=1/100;
Ts1=1/(5*200);
Ts2=.10*1/(50*200);

t1=0:Ts1:T;
t2=0:Ts2:T;

xsamples=sin(2*pi*100*t1);
x=sin(2*pi*100*t2);

figure(1);
clf;
hold on;
stem(t1,xsamples);
plot(t2,x);
title('Reproduction of Figure 6.2');
print -depsc hw6problem6figure1.eps


xrecon=zeros(1,length(t2));

for n=1:length(t1)
    temp=xsamples(n)*sinc(1000*(t2-t1(n)));
    plot(t2,temp,':');
    xrecon=xrecon+temp;

end

plot(t2,xrecon,'r');
```

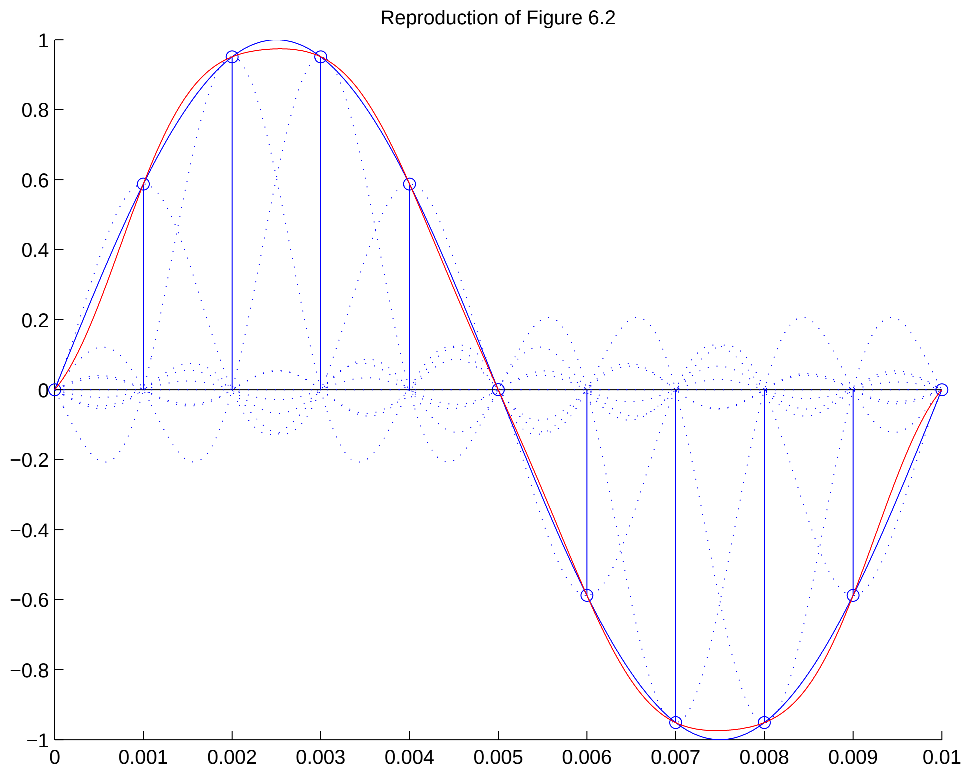
Reproduction of Figure 6.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01