EE440 HW#5 Solution

March 8, 2011

1. In class we discussed narrow band angle modulated signals (NBFM/NBPM) as the truncation of an infinite series. We justified the truncation by saying that $k_p a(t)$ was small, so the higher order terms are negligible. The bandpass signal model we discussed in class (quadrature signal model) can also be used to justify the NBFM/NBPM approximation. When we derived the in-phase and quadrature components in class we used the identity $\sin(a+b) = \sin a \cos b + \cos a \sin b$. We could have defined $I(t)$ and $Q(t)$ slightly differently and used the identity $\cos(a+b) = \cos a \cos b - \sin a \sin b$. Use this identity and the small angle approximations ($\cos(x) \approx 1, \sin(x) \approx x$ for x small) to derive form of a NBFM/NBPM signal when $k_p a(t)$ is small.
   Solution: We can write a signal $x(t) = \cos(\omega_c t + k_f a(t))$ as $x(t) = \cos(\omega_c t)\cos(k_f a(t) - \sin(\omega_c t)\sin(k_f a(t)$ via the identity $\cos(a+b) = \cos a \cos b - \sin a \sin b$. Using the small angle approximations as stated in the problem this means that $x(t) \approx \cos(\omega_c t) - k_f a(t)\sin(\omega_c t)$ which is the same approximation that we found by truncating the series.

2. Use MATLAB to plot an FM signal with $f_c = 1Hz$ and message signal $m(t) = 10t$. Use a sampling frequency of 10000Hz and plot the signal over the range $t \in [0, 3]$s. Make two plots: one using the closed form expression for the message, and one using a numerically integrated approximation of the message. (Hint: use the rectangle rule and find a MATLAB command that takes the cumulative sum of a vector!). What are the minimum and maximum instantaneous frequencies for the signal you have been working with?
   Solution: The requested signal must be of the form

$$x(t) = \cos(2\pi t + \int_{-\infty}^{t} m(\alpha)d\alpha)$$

$$= \cos(2\pi t + \int_{-\infty}^{t} 10\alpha d\alpha)$$

$$= \cos(2\pi t + 5t^2)$$

   so $\Theta(t) = 2\pi t + 5t^2$and $\frac{d\Theta}{dt} = 2\pi + 10t$. This is minimum at $t = 0$and maximum at $t = 3$. The minimum is 1Hz and the maximum is 5.776Hz. This can also be verified by looking at the plot and approximating the maximum frequency by the inverse of the period of the last cycle. Why isn't inspecting the plot completely accurate? The following MATLAB produces the required plots.

```
fs=10000;
Ts=1/fs;
t=0:Ts:3;
x1=cos(2*pi*1*t+cumsum(10*t.*Ts));
x2=cos(2*pi*1*t+5*t.^2);
subplot(211) plot(t,x1)
subplot(212) plot(t,x2)
```

3. Given a message $m(t) = 5\sin(2\pi 10^3 t)$ find the output of an FM modulator using a carrier frequency of 10^7Hz.

Solution: For FM $\phi(t) = \int_{-\infty}^{t} m(\alpha)d\alpha = 5\int_{-\infty}^{t} \sin(2\pi 10^3 \alpha)d\alpha$ this definite integral can be evaluated to find that $\phi(t) = \frac{5}{2\pi 10^3}(\cos(2\pi 10^3 \infty) - \cos(2\pi 10^3 t))$. The first term here can be chosen arbitrarily for the purposes of this problem to be any value from -1 to 1. I choose 0. So the signal $x_{FM}(t) = \cos(2\pi 10^3 t - \frac{\cos(2\pi 10^3 t)}{2\pi 10^3})$

4. Draw block diagrams which use only summing junctions, mixers/multipliers, integrators, and $\frac{\pi}{2}$ phase shifters to generate:

   (a) NBFM

   (b) NBPM
   Solution: See Figure 5.8 on page 273 of your text. Replace the DSB-SC block in each diagram with a mixer.

5. In April 1970 RCA introduced a new recording format for music, the Quadraphonic 8-Track. Unlike formats common today, this format allows for four channels of recorded music to be played. This format is matrix encoded, JVC also introduced a system using discrete encoding.

   (a) Investigate the difference between matrix encoding and discrete encoding.
   Solution: In matrix encoding the data is blended into multiple channels, discrete keeps the channels separate.

   (b) Design a system similar to broadcast FM that would allow for the broadcast of quadraphonic recordings. Assume that the FCC has mandated that you must still allow monophonic receivers to play the sum of all four channels. Do not worry about maintaining compatibility with current stereo receivers. If you come up with a method that does, you get extra credit. Document your design with block diagrams for both the transmitter and receiver. Be sure to correctly account for gains in your signal path, that is if the original channels of sound are A, B, C, and D you must recover them with the correct power levels at the receiver. Also, draw a spectral diagram of your signal format.
   Solution: I chose a scheme where the transmitted signals are as follows:

$$s_1 = A + B + C + D$$

$$s_2 = A - B + C - D$$

$$s_3 = A - C$$

$$s_4 = B - D$$

   these can be recovered

$$A = \frac{(s_1 + s_2) + 2s_3}{2}$$

$$B = \frac{(s_1 + s_2) - 2s_3}{2}$$

$$C = \frac{(s_1 - s_2) + 2s_4}{2}$$

$$D = \frac{(s_1 - s_2) - 2s_4}{2}$$

   These signals would have to be appropriately modulated and demodulated in your design. One possible design would have signals with 15kHz of bandwidth each, and would use a base pilot of 16kHz with signals 2-3 at 32 48 and 64 kHz respectively.

   (c) Although popular music formats are two channel these days, another form of entertainment comes with six channels of sound. It is called 5.1 channel something. What is the something? Why do they call it 5.1 when it has 6 channels?
   Solution:Surround sound. The sixth channel has only one tenth of the bandwidth of the other channels, hence 5.1 channel surround sound.

6. In class when I guessed where the RDS signal is placed in an FM signal I was totally wrong. Look up RDS and explain where it is located in the FM signal and what its bit rate is. Explain one or two of the services that RDS carries.
   Solution: RDS is located at a 57kHz subcarrier in the Broadcast FM message signal. This is the 3rd harmonic of the 19kHz pilot tone. The signal uses 8kHz of bandwidth and has a bit rate of 1.185 kbps. Example services include clock time and program identification.