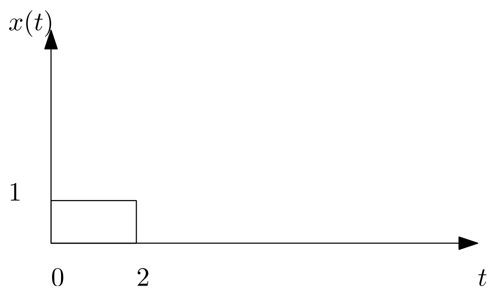
EE4440 HW#1 Solution

January 20, 2011

1. Given a signal the signal x(t) shown below in (a), plot or sketch(b-e):

   (a) 

   (b) $x(t-4)$

   (c) $x(4t)$

   (d) $x(-t)$

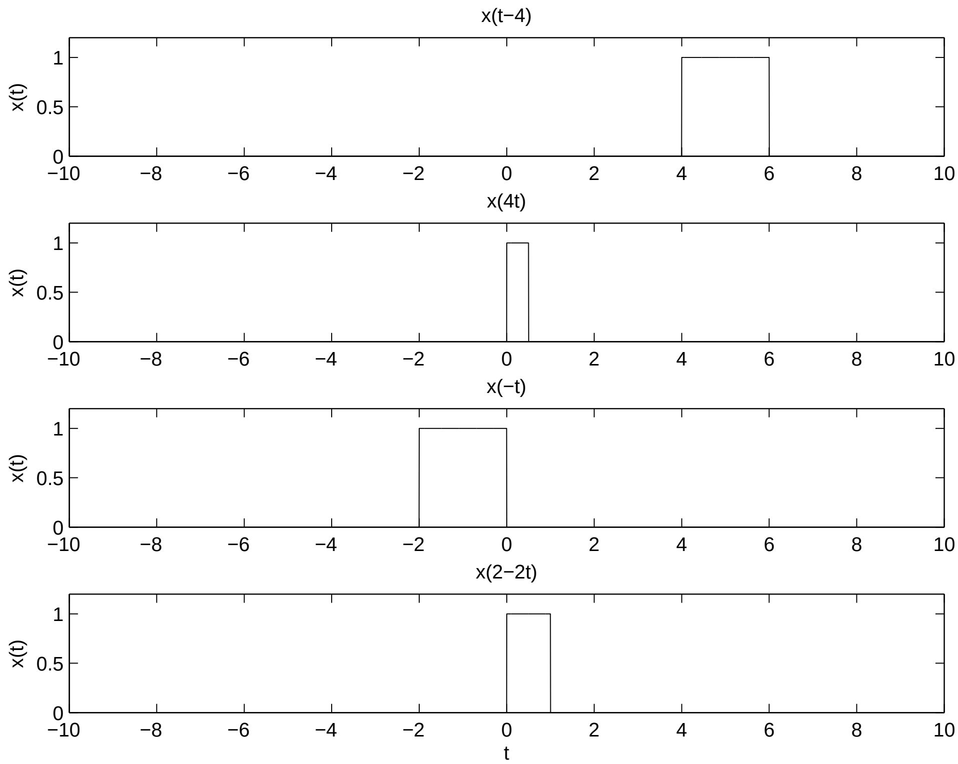
   (e) $x(2-2t)$

   Solution: The following MATLAB program solves:

```
function hw1prob1sol()

dur=10;

t=linspace(-dur,dur,5000);

subplot(411);
plot(t,rect(t-4));
axis([-dur dur 0 1.2]);
title('x(t-4)');
ylabel('x(t)');


subplot(412);
plot(t,rect(4*t));
axis([-dur dur 0 1.2]);
title('x(4t)');
ylabel('x(t)');


subplot(413);
plot(t,rect(-t));
axis([-dur dur 0 1.2]);
title('x(-t)');
ylabel('x(t)');


subplot(414);
plot(t,rect(4-2*t));
axis([-dur dur 0 1.2]);
title('x(2-2t)');
xlabel('t');
ylabel('x(t)');
```

```
print -deps 'hw1prob1figure.eps';

function y=rect(x)
    y=x>0 & x<2;
```

and the plots look like:



Note that students were not required to solve via MATLAB, this code is so you will have some idea how pulses, etc might be plotted in MATLAB. How would you plot that weird little ramp/block thing that we used in class to explain this material? Also NOTE: this can of course be solved via either method use in class: 1. Shift-Then-Scale or 2. Solve the argument for zero for key points! Keep in mind that if you solved this with MATLAB you should be sure that you can do it one or both of the other ways for exams. (That should be read as a huge hint.)

2. Answer the following

   (a) What is the primary advantage of digital communications over analog?

   Solution: That by thresholding $m(t)$ can be exactly recovered.

   (b) What two properties of channels prevent error free communication at an unlimited data rate?

   Solution: Limited bandwidth and limited SNR

3. Draw a labeled block diagram for two people communicating over a tin can telephone system. Show communication going in one direction only. If you don't know what that is look it up in Wikipedia, but don't tell me because you'll make me feel old.

   (a) Be sure to label the transmitter, receiver, source, destination, and channel.

   Solution: Should depict a person as source, a tin can as transmitter, a string as the channel, a tin can as receiver, and a second person as the destination.

(b) What kinds of noise or interference might impede communication using this system?

Solution: Anything reasonable is OK, birds landing on the string, a truck driving by, thunder, or an earthquake, enemy agents jamming the string by pulling on it, etc.

(c) By what physical mechanism is communication accomplished using this system(i.e. what physical phenomenon)?

Solution: Mechanical Vibration, bonus for mentioning longitudinal waves.

4. Assume that you have now purchased a modem for your tin can telephone. If the system has 20kHz of available bandwidth what is the channel capacity in Mbps if the SNR is:

   (a) 1dB
   (b) 3dB
   (c) 100dB
   (d) 1000dB
   (e) If you continue to increase the SNR by increasing transmit power what physical system failure will eventually come into play?

   Solution: the following MATLAB code solves:

```
SNR=[1 3 100 1000];
SNRlin=10.^(SNR/20);
B=20e3;
C=B.*log2(1+SNRlin);
C/1e6
```

   with the result:

```
[0.0217     0.0254     0.3322     3.3219] Mbps
```

   The string will eventually break because you are putting too much power into it. If you used power dB instead of amplitude dB that is OK, and you will get full credit. I didn't specify.

5. Given three signals $x(t)$,$y_1(t)$, and $y_2(t)$ defined as :

$$x(t) = \begin{cases} t & abs(t) \leq \pi \\ 0 & abs(t) > \pi \end{cases}$$

$$y_1(t) = \begin{cases} cos(t) & |t| \leq \pi \\ 0 & |t| > \pi \end{cases}$$

$$y_2(t) = \begin{cases} sin(t) & |t| \leq \pi \\ 0 & |t| > \pi \end{cases}$$

Find the dot products $< x, y_1 >$ and $< x, y_2 >$. That's right, you need to remember integration by parts. Or maybe a table would be easier? Mathematica? Anything but copying is OK, if you don't work it out by hand, please note your source or method. If you know the identity, lie and tell me some table or Calculus book where you found it.

Solution:

$$< x, y_1 >= \frac{1}{2\pi} \int_{-\pi}^{\pi} t\cos(t) = t\sin(t) + \cos(t)|_{-\pi}^{\pi} = 0$$

and

$$< x, y_2 >= \frac{1}{2\pi} \int_{-\pi}^{\pi} t\sin(t) = \sin(t) - t\cos(t)|_{-\pi}^{\pi} = 1$$

.

6. Show that $e^{jk\omega_0 t}$ and $e^{jm\omega_0 t}$are orthogonal for $k \neq m$ for an inner product defined as:

$$< f, g >= \frac{1}{T} \int_0^T f(t) g^*(t)$$

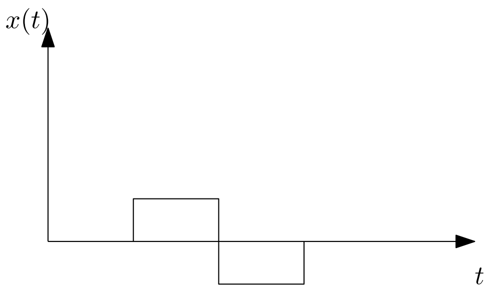
where $\omega_0 = \frac{2\pi}{T}$. Recall that $e^{j2\pi k} = 1$. A hint, this is a trivial integration so don't make this harder than it needs to be!

Solution:

$$\frac{1}{T} \int_0^T e^{jk\omega_0 t} e^{-jm\omega_0 t} dt = \frac{e^{j2\pi(k-m)} - 1}{j2\pi(k-m)} = 0$$

because $e^{j2\pi n} = 1 \forall n$and $\omega_0 T = 2\pi$.

7. For the signal x(t) shown below in (a) complete (b-d), graphically indicate that the areas work out:



(a)

(b) Sketch a best friend (signal with ρ=1)

(c) Sketch a complete stranger (signal with ρ=0)

(d) Sketch a worst enemy (signal with ρ=-1)

Solution: b should be either a repeat or scaled version, c should be orthogonal, d should be the inverted signal.

8. In example 2.12 we showed that the Fourier series coeffecients for a pulse train $g(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$ are $D_n = \frac{1}{T_0}$. That means that we can express $g(t)$as $g(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{j2\pi f_0 t}$ with $f_0 = \frac{1}{T_0}$. Use Euler's identity to write this function as a sum of real functions $\{y_0(t) + y_1(t) + y_2(t) + ... + y_N(t), ...\}$of time. Plot this expression for $N = \{3, 10, 15\}$with $f_0 = 1Hz$ in MATLAB over the time period -10 to 10 seconds. Include your MATLAB code with your homework.

Solution: the required expression is $\{1 + 2\cos(2\pi t) + 2\cos(2\pi 2t) + ...2\cos(2\pi nt) + ...$ code to solve is something like:

```
t=linspace(-10,10,1000);

subplot(311);
x=ones(1,length(t));
for m=1:3
    x=x+2*cos(2*pi*m*t);
end
plot(t,x);
title('N=3')

ylabel('Amplitude');


subplot(312);
x=ones(1,length(t));
for m=1:10
    x=x+2*cos(2*pi*m*t);
end
```

```
plot(t,x);
title('N=10');
ylabel('Amplitude');


subplot(313);
x=ones(1,length(t));
for m=1:15
    x=x+2*cos(2*pi*m*t);
end
plot(t,x);
title('N=15');
xlabel('Time(s)');
ylabel('Amplitude');

print -deps 'hw1prob8figure.eps'
```

and the plots look something like: