The University of Wyoming

College of Engineering and Applied Science

EE 4440
Communication Theory
Exam 1a, Spring 2011

Printed Name: Solution

By signing below, I attest that I am the person taking this exam and that I neither gave nor received any unauthorized assistance in completing it.

Signature: ______________________

Instructions—read carefully!

Do *not* turn this page to begin the Exam until instructed to do so by the instructor.
You have 75 minutes to complete the 6 questions on this Exam; there are a total of 100 points for the entire Exam. Point values for the questions are given in boxes at the right margin. The Exam is open-book and closed-notes except as specifically authorized. AUTHORIZED RESOURCES: a calculator, your book, and any appropriate notes of your choice on one 8×11 inch double-sided sheet of paper.
Answer the questions in this Exam in the space available. Show *all your work* used to arrive at an answer *to be eligible for partial credit.*

1. Given the two basis signals $\phi_1(t) = \Pi(t-.5)$ and $\phi_2(t) = \Pi(t-1.5)$ shown in the Figure 1:

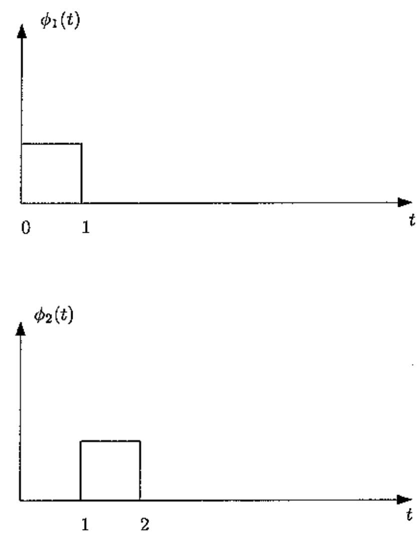


Figure 1

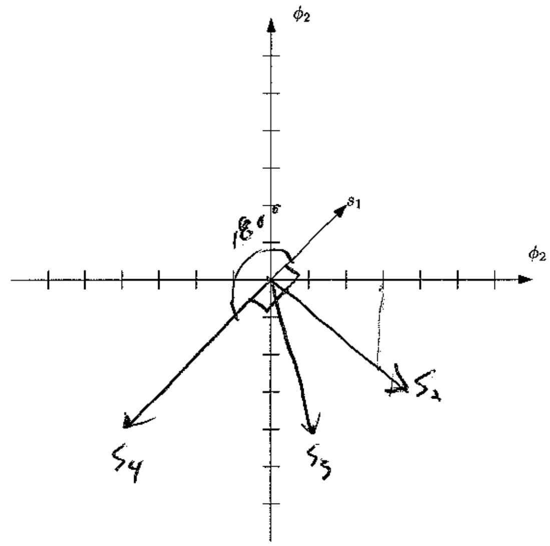


Figure 2

(a) Are ϕ_1 and ϕ_2 orthogonal functions? State in the simplest way you can think of why they are, or are not, orthogonal. $\boxed{10}$

Yes, because since the two signals don't over-lap $\int \phi_1 \phi_2 dt = 0$.

(b) Draw the coordinates (the coefficients of the basis functions) for $s_1 = (2\phi_1 + 2\phi_2)$, $s_2 = (3\phi_1 - 3\phi_2)$, $s_3 = (\phi_1 - 4\phi_2)$ and $s_4 = (-4\phi_1 - 4\phi_2)$on the axis provided in Figure 2 (I've drawn s_1 so you'll see what I want). Amongst these four signals are any pairs orthogonal? If so, what is the angle between coordinate vectors of the orthogonal signals? Note this angle in your diagram. Amongst these four signals are any pairs antipodal (worst enemies)? If so, what is the angle between coordinate vectors of the antipodal signals? Note this angle in your diagram. $\boxed{10}$

$S_2 \perp S_1, S_4 \Rightarrow 90°$ in diagram

S_1 & S_4 Antipodal $\Rightarrow 180°$ in diagram

2. Circle True or False:

   (a) $\boxed{\text{True}}$ or False : A signal that is periodic in time has a discrete frequency spectrum. [1]

   (b) True or $\boxed{\text{False}}$ Shifting a signal in time alters the magnitude spectrum of the signal. [1]

   (c) True or $\boxed{\text{False}}$ The Fourier Series exists for any real signal. [1]

   Periodic Only!

   (d) True or $\boxed{\text{False}}$ The transmission medium for radio is air, that is, in a vacuum radio waves can not be transmitted from one location to another. [1]

   Radio works in space!

   (e) True or $\boxed{\text{False}}$ It is simple to build an analog communications system that allows for perfect recovery of the message signal at the receiver. [1]

   (f) True or $\boxed{\text{False}}$ Frequency measured in rad/s (ω) is a factor of 2π smaller than the corresponding frequency in Hz (f). [1]

   $\omega = 2\pi f$ so ω 2π larger

   (g) $\boxed{\text{True}}$ or False: The signal $x(t) = \cos(\omega_0 t)$ is periodic. [1]

   (h) True or $\boxed{\text{False}}$ The signal $x(t) = \cos(\omega_0 t)$ is an energy signal. [1]

   (i) $\boxed{\text{True}}$ or False: Any real periodic signal can be written as an infinite sum of appropriately scaled complex exponential functions (such as $e^{jnw_0 t}$). [1]

   (j) True or $\boxed{\text{False}}$ The topics in this course are only useful to students who plan to become specialists in communications systems. [1]

3. Answer the following short answer questions:
   (a) Convolution in the time domain corresponds to multiplication in the frequency domain. [1]

   (b) Multiplication by a complex exponential function (e^{jwt}) in time corresponds to convolution with a delta function in frequency. [1]

   (c) The Energy Spectral Density $\Psi_x(f)$ of a signal $x(t)$ can be found by either the product $G(f)\,G(f)$ or by the Fourier Transform of ACF or $\Psi_x(f)$ [1]

   (d) If the power of a modulated signal is A, then the power of the signal PRIOR to modulation must have been 2A. [1]

   (e) A signal that has duration .001s has approximate bandwidth of 1 kHz. [1]

   (f) If a signal duration grows then the signal bandwidth must shrink. [1]

   (g) The true bandwidth of $\tau sinc(\pi f \tau)$is ∞, but a practical approximation of the bandwidth is $1/\tau$. [1]

   (h) The two conditions required for a communications channel to be considered distortion free for a particular signal are constant gain and linear phase for all frequencies contained in the bandwidth of the signal. [1]

   (i) Linear phase is important because it implies a constant delay for all frequencies of a signal. [1]

   (j) Suppose that a communications channel accommodates a bandwidth of 4kHz, a signal to noise ratio of 12.6 dB is required to allow for a channel capacity of 9600 bits per second. (Please give your answer in amplitude dB, that is $20 \log_{10} \frac{S}{N}$, and show your work below.) [1]

4. Find the Fourier Transform for the signal: $x(t) = \sin(wt) + \sin^2(\omega t)$. Use Euler's identities and table properties and please DO NOT INTEGRATE ANYTHING!!!!! [20]

$$\sin(w_0 t) = \frac{e^{jw_0 t} - e^{-jw_0 t}}{2j}$$

$$\sin^2(w_0 t) = \frac{\left(e^{jw_0 t} - e^{-jw_0 t}\right)\left(e^{jw_0 t} - e^{-jw_0 t}\right)}{-4}$$

$$= \frac{e^{j2w_0 t} - e^{0} - e^{0} + e^{-jw_0 t}}{-4}$$

So $$\boxed{X(f) = \frac{1}{2j}\left[\delta(w-w_0) - \delta(w+w_0)\right] - \frac{1}{4}\left[\delta(w-w_0) - 2\delta(w) + \delta(w+2w_0)\right]}$$

5. Given that a signal $x(t) = 10sinc(2\pi 5t)\cos(2\pi 20,000t)$ find $X(f)$ the Fourier Transform of $x(t)$ and draw a labeled sketch of $X(f)$. On the sketch, note the bandwidth of $X(f)$. $\boxed{20}$

$$x(t) = \underbrace{10\,\mathrm{sinc}(2\pi 5t)}_{m(t)}\ \underbrace{\cos(2\pi\ 20k\ t)}_{c(t)}$$

$$X(f) = M(f) * C(f)$$

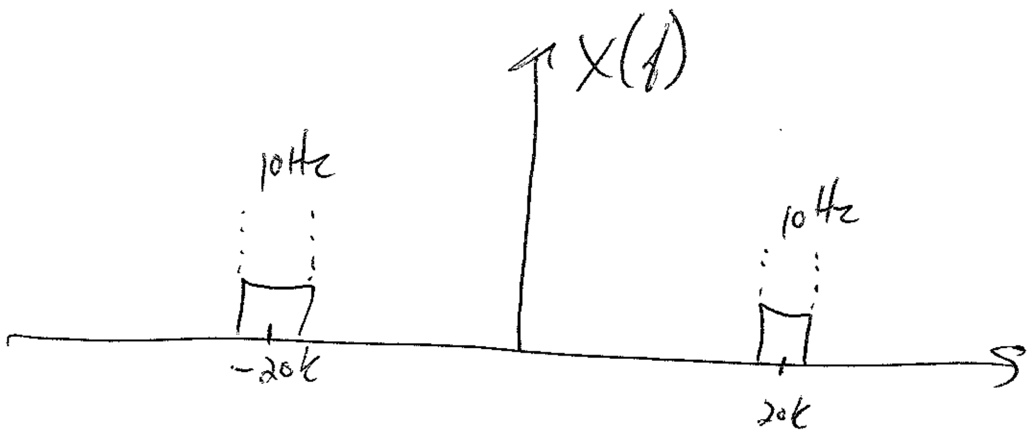
from table 3.1

$$2B\,\mathrm{sinc}(2\pi B t) \longleftrightarrow \Pi\left(\frac{f}{2B}\right)$$

so $m(t) = 10\,\mathrm{sinc}\,2\pi 5t \longleftrightarrow \Pi\left(\frac{f}{10}\right)$ spans $-5 \to 5$

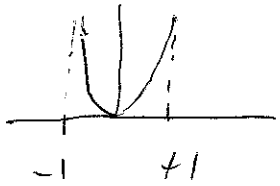
$$c(t) = \cos(2\pi\ 20{,}000\,t) \longleftrightarrow .5\left[\delta(f+20000) + \delta(f-20000)\right]$$

$$X(f) = \Pi\left(\frac{f}{10}\right) * \ .5\left[\delta(f+20k) + \delta(f-20k)\right]$$

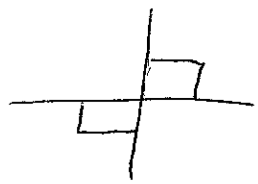


6. Given the two signals: $x(t) = t^2$ for $t \in [-1, 1]$ and $y(t) = -\Pi(t + .5) + \Pi(t - .5)$, find the correlation coefficient, ρ. Are $x(t)$ and $y(t)$ best friends, worst enemies, or complete strangers? $\boxed{20}$

$x(t) = t^2 =$



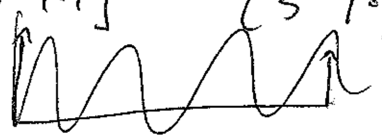
$y(t) = -\Pi(t+.5) + \Pi(t-.5) =$



$$\rho = \frac{\frac{1}{2}\int_{-1}^{1} x(t)\,y(t)\,dt}{\sqrt{E_x}\,\sqrt{E_y}}$$

$$\frac{1}{2}\int_{-1}^{1} x(t)\,y(t) = -1\int_{-1}^{0} t^2\,dt + 1\int_{0}^{1} t^2\,dt$$

$$= -1\left[\frac{t^3}{3}\Big|_{-1}^{0}\right] + \left[\frac{t^3}{3}\Big|_{0}^{1}\right]$$



$$-1\left(0 + \frac{1}{3}\right) + \left(\frac{1}{3} - 0\right) = 0$$

so $\rho = 0$ & $x(t) \perp y(t) \Rightarrow$ complete strangers