Numerical Study of the Unsteady Aerodynamics of Fluttering and Tumbling Cards

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Abstract

The fluttering of a leaf, the free fall of an object and the autorotation of a windmill are few of the many unsteady phenomena involving a fluid - body interaction that has served as an inspiration for scientists to develop and use efficient computing tools for solving fluid dynamics problems. This paper addresses these phenomena from a computational point of view. Solutions to the unsteady incompressible Navier Stokes equations coupled with rigid body dynamics on moving overlapping meshes are obtained at low Reynolds numbers. Computations are carried out to verify existing experimental observations, numerical results and to provide more insight into understanding fluttering and tumbling of rectangular cards. A fast Fourier transform of the aerodynamic torque is computed and it provides some very interesting features about these motions. Based on computed results, an approximate mathematical model for torque is formulated, which enables us to find relationships between frequency of oscillation and the frequency of vortex shedding. Falling cards in two- and three-dimensions have also been simulated and their sensitivity to moment of inertia analyzed.

Keywords: Unsteady Flows, Overlapping Mesh, flutter, autorotation

1 Introduction

The free fall of objects under the action of gravitational force has long been admired by scientists and researchers worldwide owing to the manner in which it falls and displays some fascinating flow phenomena. The dynamics is sometimes unpredictable and chaotic, which sometimes renders lower order computational models to fail. Earlier experiments conducted by Maxwell [1] in 1853 were primarily motivated by the need to understand these dynamics. Maxwell’s explanation for the cause of autorotation in which a falling plate attains the state of continuous rotation was rectified by the work of Riabouchinsky [2] where he emphasizes the main differences between fixed and freely moving axes. Autorotation of wings formed a major portion of the study during the first half of the twentieth century with its application to control
of spinning finned missiles, bomblets and seeds [3]. A variety of experiments on plates which oscillate about an axes perpendicular to the direction of motion were conducted. Willmarth et al [4] provided some impressive results through experiments on the fall of a circular disk and introduced the concept of dimensionless moment of inertia ($I^*$) and its effect on the transition between side to side oscillations and tumbling motions. Further literature on the development of early experimental models can be found in Mahadevan et al [5]. A good amount of numerical computations relating to this topic can be found in Tanabe and Kaneko [6], Mittal et al [7], Pesavento and Wang [8], Jones A.M et al [9], Saephen and Van Dam [10] and Murman et al [11]. Extensive experiments on falling plates, cylinders can be found in Belmonte et al [12], Chu et al [13] and Field et al [14]. The present research adds more flavor to this understanding by describing the circumstances in which a body exposed to a uniform stream of fluid can maintain itself in three different states of motion namely (a) The state of continuous rotation (Autorotation) (b) The state of high frequency oscillation (Flutter) and (c) The state of low frequency oscillation. Computations are performed on moving overlapping meshes using the object oriented code *OverBlown* [15] which is based on the *Overture* [16] framework for solving the incompressible Navier-Stokes equations.

2 Governing equations and numerical method

The motion of a freely falling card is governed by the unsteady incompressible Navier Stokes equations coupled with the dynamical equations of motion (Newton - Navier Stokes equations). The complete description of the numerical method, implementation of the boundary conditions, interpolation on the overlapping mesh and convergence studies, can be found in Henshaw [15][16], Chandar and Damodaran [17]. However for the sake of completeness, an outline of the numerical method is provided here. The time dependent incompressible Navier Stokes Equations are given by,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{\nabla p}{\rho} = \nu \Delta \mathbf{u} + \mathbf{F}$$  \hspace{1cm} (1)$$

$$\nabla \cdot \mathbf{u} = 0$$  \hspace{1cm} (2)

Here $\mathbf{u}$ is the velocity vector, $p$ is the pressure, $\mathbf{F}$ is a function of external forces, $\nu$ is the kinematic viscosity and $\rho$ is the fluid density. These equations are discretized in space on a system of overlapping meshes. An implicit multistep method is used for time stepping and second order differences are used for spatial discretization. The pressure is obtained by solving a pressure poisson equation using the yale direct sparse matrix package in Overture [16] for two-dimensional problems and using the parallel implementation of Krylov subspace methods (Bi-Conjugate Gradient Stabilized) from PETSc(Portable Extensible Toolkit for Scientific Computation) [18] for three-dimensional problems. The acceleration of the body is computed using aerodynamic forces $F_A$ and torques $T = \int d\Omega (r - x_{cm}) \times dF_A$ where $r$ is any point on the body and $x_{cm}$ is the center of mass. This appears in the boundary condition for pressure in the pressure equation. The Navier-Stokes equations are solved with this boundary condition and a new set of forces are obtained. This procedure is repeated over a period of time till the
final time of computation. When the meshes move rigidly with the body, the solutions in the region of overlap are interpolated using a Lagrange interpolation formula. Apart from Reynolds number, there are other non-dimensional quantities like the moment of inertia and Froude number that govern these type of unsteady flows. In the present study, a rectangular plate of length \( L \), width \( w \), thickness \( h \) and density \( \rho_b \) moves either freely under the action of gravitational force \( mg \) or rotates about a point fixed to the center of mass in a fluid whose density is \( \rho_f \). In two-dimensions, the dimensionless moment of inertia and the Froude number are given by

\[
I^* = \frac{I}{\rho_f L^4} = \frac{1}{12} \frac{\rho_b h}{\rho_f} \left( \frac{h^2}{L^2} + 1 \right) = \frac{1}{12} \zeta \beta \left( \beta^2 + 1 \right), \quad Fr^2 = \frac{M}{\rho_f L^2} = \frac{\rho_b h L}{\rho_f L^2} = \zeta \beta \tag{3}
\]

where the moment of inertia \( I \) about the axis of rotation has been non-dimensionalized by a quantity proportional to the moment of inertia of the displaced fluid [4]. However the term \( \beta^2 + 1 \) is not included in the computations of Mittal et al[7]. From the above equation, the relationship between \( I^* \) and \( Fr \) is evident. i.e., \( I^* \propto Fr^2 \) which has also been stated in Pesavento and Wang [8], Mittal [7]. The Reynolds number \( Re \) is calculated based on a reference velocity which, for a falling plate is the terminal velocity \( U_T = \sqrt{2g (\zeta - 1) h} \) and for a pinned plate, a specified freestream velocity.

### 3 Results and discussions

Various code validation studies for unsteady flows can be found in Chandar and Damodaran [17]. An additional validation test case is reported in this paper since it pertains to autorotation of pinned plates and is relevant to the present study. The computational test case of Lugt [3] is used to demonstrate the concept of autorotation. The forced rotation of an elliptic cylinder is considered at different spin parameters \( S = \bar{\omega}L/2U \) where \( \bar{\omega} \) and \( U \) are the angular velocity of the plate and the free stream velocity respectively. The Reynolds number based on chord and free stream velocity is 200. The computational mesh system consists of two overlapping meshes (a cartesian background mesh and a curvilinear mesh for the cylinder) as shown in Fig. 1(a). The cartesian background mesh and the curvilinear mesh are of sizes 100 \( \times \) 100 and 120 \( \times \) 70 respectively. For \( S = 0.25 \), when the cylinder is inclined at 135\(^\circ\) to the horizontal, the vortic-

![Figure 1: (a)The overlapping mesh system consisting of a background cartesian mesh and a curvilinear mesh (b) Computed vorticity contours from Lugt[3] (c) Computed vorticity contours using OverBlown (d) Computed \( \Pi \) contours using OverBlown](image-url)
ity and Π contours[20] \((Π = min \left( \hat{S}^2 + Ω^2, 0 \right) \)) are plotted along with the results of Lugt[3] in Fig. 1(b)-1(d). Here \(\hat{S}\) and Ω are the symmetric and anti-symmetric parts of the dyad \(\nabla u\). The vortex cores are easily identified in Fig. 1(d). The moment coefficient at various angles of attack is plotted in Fig. 2(a)-2(d) and shows a good agreement with the results of Lugt [3]. Also plotted in Fig. 3(a)-3(b) are the variation of lift and drag coefficients over half a cycle of rotation. The results are compared with the computed results of Lugt [3] and the experimental results for an autorotating wing at \(Re = 90000\) from Smith [19] \((β = 0.15, S ≈ 0.8 – 1.0)\). The computations confirm the observation of Lugt [3] that the Reynolds number has little influence on the aerodynamic force coefficients in the range \(200 ≤ Re ≤ 90000\).

Figure 2: Computed moment coefficients at various angular positions for various spin parameters in comparison with the computed results of Lugt [3]

Figure 3: Computed lift and drag coefficients over half a cycle of rotation in comparison with the computed results of Lugt[3] for Re=200 and the experimental results of Smith[19] for Re=90000

3.1 Fluttering and tumbling of a rectangular card

The computational cases considered by Mittal et al [7] are revisited. It is seen that there is a difference between classical flutter and flutter as described by Mittal et al [7]. A series of computations at \(Re = 300\) involving rectangular cards of varying thickness which are pinned to its center of mass and free to oscillate by virtue of the aerodynamic forces are considered. The time history of the lift coefficient for case (a)\(β = 0.5\), case (b)\(β = 0.2\) are shown in Fig. 4(a)-4(b) along with the computations of Mittal et al [7]. Case (a) was classified as fluttering mode whereas case (b) was classified as autorotative mode. The comparison being satisfactory, further experiments were performed to understand the flutter - autorotation/tumbling
transition. Numerical experiments for very small thickness ratios $< 0.2$ revealed that there exists another mode of oscillation which is the correct flutttering mode. This type of flutter is accompanied by high frequency oscillations. Fig. 4(c) shows the time history of the card angular velocity for different thickness ratios. It is clearly observed that for low thickness ratios, flutter is observed and as the thickness ratio is increased, the body starts to autorotate and then a further increase in thickness ratio renders the body to oscillate at low amplitude and frequency (not flutter as pointed out by Mittal et al [7]). Computations have also been performed for thickness ratios $\beta = 0.05, 0.1, 0.2, 0.4, 0.5, 0.7, 0.75, 1.0$ and density ratios $\zeta = 3.5, 4.08, 7.0, 10.2, 20.0, 40.0, 80.0, 100.0$. Based on this, a plot of $I^*$ vs $\beta$ for different $\zeta$ is obtained as shown in Fig. 5(a). It is very clear from this plot that a body which is pinned to its center of mass and excited by aerodynamic forces will eventually attain any one of the three different states of motion (flutter / autorotation / low frequency oscillation). When the density and thickness of the body is high, the body has high inertia and rotates slowly. The frequency of vortex shedding will be higher than the frequency of rotation and will act to oppose the motion of the body. Multiple harmonics are present in the time history of torque due to the fact that vortices can be shed from every corner of the body. See Fig 5(b)-5(c) for example. This is also evident from the results of the previous section, Fig. 2(a)-2(d). For a narrow range of thickness ratios, it is seen that for every half a cycle of rotation, one vortex pair is shed. In this range, the body attains the state of autorotation. The multiple harmonics tend to subside but are still present. For low density and thickness ratios, the body has very little inertia and its motion is highly influenced by the shedding of vortices. The body oscillates violently with a very high frequency and the time history of torque shows a single frequency. It can also be concluded that, for low Reynolds numbers, a square card will never flutter.

3.1.1 FFT analysis and a mathematical model for torque

From the time history of torque for thickness ratio $\beta = 0.1$ as seen from Fig. 5(b)-5(c), it is very difficult to conclude exactly how many frequency components are present and what the individual frequencies are. Hence a fast Fourier transform (FFT) of the torque waveform is computed for all thickness ratios and densities. These results are shown in Fig. 6(a)-6(e). A very convincing observation can be made by analyzing these data. The points $(\beta, \zeta)$ corresponding to a single peak are the ones that flutter. This is also verified from the phase plot of Fig. 5(a). Both
Figure 5: (a) $I^*, \beta$ phase plot showing the transition between flutter, autorotation and low frequency oscillations (b) Time history of torque for large density ratios (c) Time history of torque for small density ratios

(a) $\beta = 0.1$  (b) $\beta = 0.2$  (c) $\beta = 0.5$

(d) $\beta = 0.75$  (e) $\beta = 1.0$

Figure 6: Power spectrum of torque for various thickness ratios

autorotation and low frequency oscillations do have multiple frequency modes and this confirms our observation about the existence of multiple modes from the previous section. What exactly do these frequencies represent is a question of interest. To understand this further, a series of computations for a fixed card with its broadside normal to the flow have been performed and the vortex shedding frequencies computed for each of the thickness ratios. These frequencies are circled in Fig. 6(a)- 6(e). A very important conclusion can be drawn by examining carefully the density ratios corresponding to the circled positions. It can be concluded that when a body is executing low amplitude oscillations (not flutter nor autorotation), one of the frequency component happens to be the natural frequency of vortex shedding $\omega_n$ in its stable position. If the card has a probability to flutter for a given density ratio, its frequency will be greater than $\omega_n$. 
One way of analyzing flutter, tumble or low frequency oscillations would be to use bifurcation theory to estimate the unstable point. This can be achieved by expressing the equations of motion as $I\dot{\Omega} = f(\Omega, \theta)$, $\dot{\theta} = \Omega$ and then finding the fixed points of the above ODE to check for the sign of the eigen values at fixed points. But this requires formulating the function $f$ which can be achieved through quasi-steady approximations [8].

A two frequency model for torque is proposed and is given by

$$T = a_1 \sin(k\omega t) + a_2 \sin(\omega t + \phi)$$ (4)

In the above equation, $k$ can be thought of as a quantity that is proportional to the number of times the card is in the vicinity of a vortex. $\phi$ can be a function of the time between successive vortex interactions and $a_1, a_2$ are arbitrary amplitudes. The aim is not to fit the above equation with the computed results but to give a feel of how the quantities like phase and amplitudes are related for low frequency oscillations. Eq. 4 can be integrated twice assuming zero external angular velocity at $t = 0$ to yield

$$\theta(t) = \theta_0 + \frac{1}{I\omega} \left( \frac{a_1}{k} + a_2 \cos \phi \right) t - \frac{a_1}{I\omega^2 k^2} \sin(k\omega t) - \frac{a_2}{I\omega^2} \left[ \sin(\omega t + \phi) - \sin \phi \right]$$ (5)

For flutter or low frequency oscillation to occur, $\theta(t)$ is bounded. From Eq. 5, this requires,

$$\cos \phi = -\frac{1}{k} \frac{a_1}{a_2}$$ (6)

Further, the mean angular velocity of the card $\bar{\Omega}$ should vanish over one cycle. The mean angular velocity of the card is got by integrating Eq. 4 once over one cycle. This gives

$$\bar{\Omega} = \frac{1}{I\omega} \left( \frac{a_1}{k} + a_2 \cos \phi \right) - \frac{a_1}{2\pi I\omega k^2} \sin(2\pi k)$$ (7)

Using Eq. 6 in Eq. 7, requires $2\pi k = n\pi$ or $k = n/2$. Excluding the case when $n = 2$ which corresponds to flutter, for low frequency oscillations, $k = 1, \frac{3}{2}, 2, \frac{5}{2}, ...$. From the previous section, it was concluded that one of the frequency components for a card oscillating with a low frequency is the natural vortex shedding frequency $\omega_N$. Hence by the model described by Eq. 4, $k\omega = \omega_N$ or $\frac{\omega}{\omega_N} = \frac{1}{k}$. A detailed investigation reveals that at least one of the frequencies in the lower end of the spectrum (which corresponds to the angular frequency of the body $\omega$) given in Fig. 6(a)-6(e) for the case of low frequency oscillation does in fact satisfy the above condition. This observation can be made easily for cases with higher thickness ratios since it can easily oscillate with a low frequency. From this, it can be concluded that a simple mathematical model in conjunction with an FFT can provide lot of insight into the behavior of unsteady motion of cards. A more generalized expression involving Fourier series could be adopted for torque and this could provide more understanding on the relationship between other frequencies.

### 3.1.2 Falling of a rectangular card

The free fall of a rectangular card in two- and three-dimensions has been simulated. The main difference between a falling card and a pinned card is that the axes is free to move in the former
case whereas it is fixed in the latter case. However the basic mechanism of flutter or autorotation still holds good here. In two-dimensions, a simple falling card is studied. From the phase plot Fig. 5(a), a point \((\beta, I^*) = (0.1, 0.0858)\) is picked corresponding to a flutter/autorotation transition case. This corresponds to a Froude number of \(F_r = 1.02\). For this combination of thickness ratio \(\beta\) and \(I^*\), the card with a pinned center of mass initially begins to rotate continuously but sets into a fluttering motion. However in this case, the card starts to rotate continuously and does not flutter as shown in Fig. 7(a),7(d). This is expected since the card

![Figure 7: Trajectory of the falling card and a vortex snapshot for different thickness ratios](image)

![Figure 8: (a) Vortices left behind a falling card - Z Component of vorticity (b) Trajectory in a three dimensional space (c) The card at different instances of times](image)
is hardly influenced by its shed vortices. Fig. 7(b)-7(f) show the corresponding path and vortex snapshots for different sets (0.2, 0.1768), (0.5, 0.5312). In three-dimensions, a card with a rectangular planform (Aspect ratio =2) and elliptical cross-section with 10% thickness is considered. The Reynolds number based on the estimated terminal velocity is 424. Fig. 8(a) shows the Z component of vorticity on a plane. The vortices shed by the falling card have a larger structure owing to low Reynolds numbers. Fig. 8(b),8(c) shows the path of the card in three dimensions. The card is seen to execute very similar motions compared to two-dimensional test cases. More work would be needed to quantify the effect of aspect ratio on the dynamics of free fall in three-dimensions. Nevertheless, these computations do give a fair idea of how vortex interactions can influence the behavior of a body in low Reynolds numbers.

**Conclusions**

The unsteady aerodynamics of fluttering and tumbling cards have been investigated for a large number of cases by solving the Navier-Stokes equations on using moving overlapping meshes. Various modes of oscillation have been examined and the differences between flutter and low frequency oscillation highlighted for pinned cards. A phase diagram depicting the flutter-autorotation-low frequency oscillation transition is obtained for a large number of density ratios. FFT analysis of torque in conjunction with a mathematical model provided more insights into the various frequencies present for an oscillating card and its relationship with the vortex shedding frequency. Further on, falling cards in two- and three-dimensions have been simulated for a limited number of parameters. The trajectories obtained do show a similar dependence on the thickness ratios compared to pinned configurations.

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**References**


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