Fundamental Domains of Some Drinfeld Modular Curves

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Abstract. We construct fundamental domains for arithmetic subgroups of $\Gamma = \mathrm{GL}_2(\mathbb{F}_q[t])$. Given $\Delta \supseteq \Gamma(\mathfrak{a})$ we construct a contracted form $\overline{\mathcal{T}}$ of the Bruhat-Tits tree $\mathcal{T}$ and a fundamental domain $\overline{\mathcal{F}}$ of Δ acting on $\overline{\mathcal{T}}$. We define a lift of $\overline{\mathcal{F}}$ to $\mathcal{F} \subset \mathcal{T}$ called the "bipartite" lift. We show that $\mathcal{F}$ is a fundamental domain of Δ acting on $\mathcal{T}$ precisely when $\overline{\mathcal{F}}$ is "Δ-compressed."

1. Introduction

Fix a finite field $k = \mathbb{F}_q$, let $A = k[t]$ and $K = k(t)$. We denote the ∞-adic completion of K by K_∞.

The arithmetic group $\Gamma = \mathrm{GL}_2(A)$ acts on the Bruhat-Tits tree $\mathcal{T}$ and the quotient $X = \Gamma\backslash\mathcal{T}$ is a half-line. Given a proper ideal $\mathfrak{a} \subset A$ one may consider the arithmetic subgroup $\Gamma(\mathfrak{a}) \subset \Gamma$ with the induced action on $\mathcal{T}$. Similarly one may consider an arbitrary arithmetic subgroup $\Delta \supseteq \Gamma(\mathfrak{a})$. The quotient $X_\Delta = \Delta\backslash\mathcal{T}$ is a connected graph which is the union of a finite graph X_Δ° and finitely many cusps. The genus of the finite graph and the number of cusps have been computed by Gekeler.

We say that a subtree $\mathcal{F} = \mathcal{F}(\Delta) \subset \mathcal{T}$ is a fundamental domain for Δ if the image of $\mathcal{F}$ surjects onto X_Δ and is a bijection of edges. A fundamental domain always exists and we may assume without loss of generality that it is connected. Given Δ we show how to compute a fundamental domain.

2. Properties of Connected Fundamental Domains

In this section we study properties satisfied by any (connected) fundamental domain $\mathcal{F}$ for $\Gamma(\mathfrak{a})$. We assume throughout that $\mathfrak{a} \subset A$ is a proper ideal. We start by recalling the construction of a fundamental domain of Γ (cf. section II.1.6 of [**S**]). Let $\mathcal{O} \subset K_\infty$ be the valuation ring; the valuation of $f \in K^\times$ is $-\deg(f)$.

Fix $V = K_\infty^2$ and let $\{e_1, e_2\}$ denote the canonical basis. For $n \geq 0$, let v_n denote the vertex in $\mathcal{T}$ corresponding to the lattice $\mathcal{O}t^n e_1 \oplus \mathcal{O}e_2$. The subtree $\mathcal{F}(\Gamma) \subset \mathcal{T}$

spanned by $\{v_n\}_{n\geq 0}$ is a half-line starting at v_0. Let

$$\Gamma_0 = \mathrm{GL}_2(k) \subset \Gamma, \qquad \Gamma_n = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \in \Gamma : a, d \in A^\times, \deg(b) \leq n\} \right\}, \quad n \geq 1.$$

We recall without proof the following lemma from [*loc. cit.*].

Lemma 2.0.1.

(1) *The v_n are pairwise Γ-inequivalent.*
(2) *Γ_n is the stabilizer of v_n in Γ.*
(3) *Γ_0 acts transitively on the set of edges starting at v_0.*
(4) *For $n \geq 1$, Γ_n fixes the edge from v_n to v_{n+1} and operates transitively on the remaining edges from v_n.*

Proof. This is proposition 3 on page 119 of [*loc. cit.*]. □

The lemma implies that $\mathcal{F}(\Gamma)$ is a fundamental domain of Γ (cf. corollary to proposition II.3 of [*loc. cit.*]).

Definition: Let $v \in \mathcal{T}$ be a vertex and Γ_v denote the stabilizer of v in $\mathcal{T}$. We say that v is of *type n* if $v = \gamma v_n$ for some $\gamma \in \Gamma$, in which case $\Gamma_v = \gamma \Gamma_n \gamma^{-1}$.

More generally, for an arithmetic subgroup $\Delta \supseteq \Gamma(\mathfrak{a})$, we denote the stabilizer of v by Δ_v. Let $\mathcal{T}_n \subset \mathcal{T}$ be the vertices of type n.

Corollary 2.0.2. *Suppose $n \geq 1$ and $v \in \mathcal{T}_n$. Then Γ_v acts transitively on the edges from v to $\mathcal{T}_{n-1}$. It fixes the unique edge from v to $\mathcal{T}_{n+1}$.*

Proof. The follows from part 4 of lemma 2.0.1 and the identification of Γ_v above. □

For every $n \geq 0$, $\mathcal{T}_n$ is closed under the action of Γ, a fortiori under the action of Δ. We denote the subtree of all vertices of type at most n by $\mathcal{T}_{\leq n}$, and similarly for $\mathcal{T}_{\geq n}$. They are also closed under the action of Γ.

Lemma 2.0.3. *For every $n \geq 0$, the quotient $\Gamma(\mathfrak{a})\backslash\mathcal{T}_{\leq n}$ is a finite graph.*

Proof. By definition $\Gamma\backslash\mathcal{T}_{\leq n}$ is the finite subgraph of $\Gamma\backslash\mathcal{T}$ spanned by the first $n+1$ vertices. It is a quotient of the graph $\Gamma(\mathfrak{a})\backslash\mathcal{T}_{\leq n}$ by the finite group $\Gamma(\mathfrak{a})\backslash\Gamma$, hence must be finite. □

We define the cusps of $X_\mathfrak{a} = \Gamma(\mathfrak{a})\backslash\mathcal{T}$ to be the infinite half-lines such that the origin of each is connected to at least three vertices in $X_\mathfrak{a}$ and every other vertex in the cusp is connected to exactly two. We denote the complement $X_\mathfrak{a}^\circ$ and call it the finite part.

Fix a fundamental domain $\mathcal{F} = \mathcal{F}(\Gamma(\mathfrak{a})) \subset \mathcal{T}$ of $\Gamma(\mathfrak{a})$ and let $\mathcal{F}_n$ denote $\mathcal{F} \cap \mathcal{T}_n$. We assume without loss of generality that $\mathcal{F}$ is connected. We define the cuspidal part of $\mathcal{F}$ and the finite part $\mathcal{F}^\circ$ to be the inverse image of the cusps of $X_\mathfrak{a}$ and $X_\mathfrak{a}^\circ$ respectively.

COROLLARY 2.0.4. *There exists $N \geq 0$, such that for every $n \geq N$, $\mathcal{F}^\circ \subset \mathcal{T}_{\leq n}$ and every cusp intersects $\mathcal{T}_n$ in at least one point.*

PROOF. The first part is trivial because $\mathcal{F}^0$ is a finite graph. Let $\mathcal{L} \subset \mathcal{F}$ be a cusp. By definition, $\mathcal{L}$ is an infinite graph which maps injectively to the quotient $\Gamma(\mathfrak{a})$, and hence by the previous lemma, it has finite intersection with $\mathcal{F}_{\leq N}$ for every $N \geq 0$. It is also connected, so intersects $\mathcal{F}_n$ for every $n \gg 0$. $\square$

We will see below that one may take $N = d - 1 = \deg(\mathfrak{a}) - 1$ and that for every $n \geq N$ each cusp will intersect $\mathcal{F}_n$ in exactly one point.

For any v in $\mathcal{T}$ of positive type, part 4 of lemma 2.0.1 implies that there is a unique edge from v to $\mathcal{T}_{n+1}$. We denote the edge $\varepsilon(v)$ and the terminating point $\tau(v)$.

LEMMA 2.0.5. *Suppose $n \geq 0$ and $v \in \mathcal{T}_n$.*

(1) *If $n < d$, then edges from v are $\Gamma(\mathfrak{a})$-inequivalent.*
(2) *If $n \geq d$, the stabilizer of v in $\Gamma(\mathfrak{a})$ fixes $\varepsilon(v)$ and acts transitively on the remaining edges from v.*

PROOF. Let $\gamma \in \Gamma$ be an element such that $v = \gamma v_n$. Then $\Gamma_v = \gamma \Gamma_n \gamma^{-1}$, so

$$\Gamma(\mathfrak{a})_v = \Gamma_v \cap \Gamma(\mathfrak{a}) = \gamma \Gamma_n \gamma^{-1} \cap \Gamma(\mathfrak{a}) = \gamma(\Gamma_n \cap \Gamma(\mathfrak{a}))\gamma^{-1}.$$

The last equality holds because $\Gamma(\mathfrak{a})$ is a normal subgroup of Γ. Therefore, we may conjugate by γ^{-1} and reduce to the case $v = v_n$. In this case, the lemma follows from lemma 2.0.1. $\square$

The last property imposes a strong uniformity on $\mathcal{F}_n$ for $n \geq d$.

LEMMA 2.0.6. *Suppose $n \geq d$ and $v \in \mathcal{F}_n$. There are exactly two edges in $\mathcal{F}$ from v, one of which is $\varepsilon(v)$.*

PROOF. Part 4 of lemma 2.0.5 implies that for every $n \geq d$ and $v \in \mathcal{F}_n$, there is at most one edge from v to $\mathcal{F}_{n-1}$. For a fixed v, this implies every geodesic of length $m \geq 1$ in $\mathcal{F}$, starting at v and going through $\tau(v)$, must terminate at $\tau^{(m)}(v)$. Hence there must be at least one edge in $\mathcal{F}$ from v to $\mathcal{F}_{n-1}$, otherwise $\mathcal{F}$ would not be connected. It remains to show that $\varepsilon(v)$ is in $\mathcal{F}$.

Because $\mathcal{F}$ is a fundamental domain, there are a unique edge e in $\mathcal{F}$ and some $\gamma \in \Gamma(\mathfrak{a})$ such that $\gamma e = \varepsilon(v)$. From the above, there is a unique edge e' in $\mathcal{F}$ from $v' = \gamma^{-1} v$ to $\mathcal{F}_{n-1}$. Also, $\gamma e'$ is the unique edge in $\mathcal{F}$ from v to $\mathcal{F}_{n-1}$. Because $\mathcal{F}$ is a fundamental domain, we must have $\gamma e' = e'$, which implies $v' = \gamma v' = v$. Hence $\varepsilon(v) = \varepsilon(v') = e$ is in $\mathcal{F}$. $\square$

COROLLARY 2.0.7. *For every $n \geq d$ and $v \in \mathcal{F}_n$, the half-line in $\mathcal{T}$ spanned by $\{\tau^{(m)}(v)\}_{m \geq 0}$ is contained in a unique cusp $\mathcal{L} \subset \mathcal{F}$. In particular, there is a one-to-one correspondance between elements of $\mathcal{F}_n$ and cusps.*

The proof is straightforward.

Theorem 2.0.8. *Every cusp $\mathcal{L} \subset \mathcal{F}$ attaches to $\mathcal{F}^\circ$ in $\mathcal{F}_{d-1}$. Moreover, if $d > 1$, there is a one-to-one correspondance between elements of $\Gamma(\mathfrak{a})\backslash\mathcal{F}_{d-1}$ and cusps. For $d = 1$, there is a unique point $v \in \mathcal{F}_0$ and a one-to-one correspondance between elements of $\mathcal{F}_1$ and cusps.*

Proof. By corollary 2.0.7, every $v \in \mathcal{F}_d$ is contained in a unique cusp $\mathcal{L} \subset \mathcal{F}$. Further, there is a unique edge starting at v and ending in $\mathcal{F}_{n-1}$, hence $\mathcal{L}$ connects to $\mathcal{F}^\circ$ in $\mathcal{F}_{\leq d-1}$. On the other hand, part 3 of lemma 2.0.6 implies that for every $v' \in \mathcal{F}_{d-1}$ there are at least three $\Gamma(\mathfrak{a})$-inequivalent edges e_i of $\mathcal{F}$ and elements $\gamma_i \in \Gamma(\mathfrak{a})$, such that $\{\gamma_i e_i\}$ are $\Gamma(\mathfrak{a})$-inequivalent edges from v'. Hence $\mathcal{L}$ must attach to $\mathcal{F}^\circ$ in $\mathcal{F}_{d-1}$.

For $d = 1$ and $v \in \mathcal{F}_0$, every edge starting at v terminates in $\mathcal{F}_1$ and is contained in a unique cusp $\mathcal{L} \subset \mathcal{F}$. Hence for every $v' \in \mathcal{F}$ distinct from v, the segment in $\mathcal{F}$ connecting v to v' is contained in a unique cusp $\mathcal{L} \subset \mathcal{F}$. In particular, v' is of positive type, hence $\mathcal{F}_0 = \{v\}$. This implies there is a one-to-one correspondance between cusps $\mathcal{L} \subset \mathcal{F}$ and edges starting at v, which in turn correspond bijectively to elements of $\mathcal{F}_1$.

Finally, suppose $d \geq 1$. From the above, distinct cusps attach at $\Gamma(\mathfrak{a})$-inequivalent points of $\mathcal{F}_{d-1}$. Conversely, let $v \in \mathcal{F}_{d-1}$. Because $\mathcal{F}$ is a fundamental domain there is a $\gamma \in \Gamma(\mathfrak{a})$ such that $e = \gamma^{-1}\varepsilon(v)$ is contained in $\mathcal{F}$. Further, e is contained in a unique cusp $\mathcal{L} \subset \mathcal{F}$ which attaches to $\mathcal{F}^\circ$ at γv. Therefore cusps $\mathcal{L} \subset \mathcal{F}$ correspond bijectively to elements of $\Gamma(\mathfrak{a})\backslash\mathcal{F}_{d-1}$. □

Remark: From the proof we see, for $d = 1$, that $\mathcal{F}$ is the union of $q + 1$ cusps all starting at the same $v \in \mathcal{T}_0$.

Let $\Delta \supset \Gamma(\mathfrak{a})$ denote an arithmetic subgroup and $\mathcal{F}(\Delta) \subset \mathcal{T}$ a connected fundamental domain of Δ acting on $\mathcal{T}$. We may assume without loss of generality that $\mathcal{F}(\Delta) \subset \mathcal{F} = \mathcal{F}(\Gamma(\mathfrak{a}))$ and view it as a fundamental domain of $\Gamma(\mathfrak{a})\backslash\Gamma$ acting on $\mathcal{F}$.

Corollary 2.0.9. *Every cusp $\mathcal{L} \subset \mathcal{F}(\Delta)$ attaches to the finite part $\mathcal{F}(\Delta)^\circ$ at a vertex in $\mathcal{F}(\Delta)_{\leq d-1}$. In particular, for every $n \geq d$ and $v \in \mathcal{F}(\Delta)_n$, there are precisely two edges from v in $\mathcal{F}$, one of which is $\varepsilon(v)$.*

Proof. The finite group $\Gamma(\mathfrak{a})\backslash\Delta$ permutes the cusps of $\mathcal{F} = \mathcal{F}(\Gamma(\mathfrak{a}))$. Hence every cusp $\mathcal{L} \subset \mathcal{F}(\Delta)$ contains a unique cusp of $\mathcal{F}$ and is disjoint from the remaining cusps. Theorem 2.0.8 implies $\mathcal{L}$ attaches to $\mathcal{F}(\Delta)^\circ$ in $\mathcal{F}(\Delta)_{\leq d-1}$. □

3. Bipartite Trees

Let $\mathcal{T}$ be the Bruhat-Tits tree. Throughout we let $\mathcal{F} \subset \mathcal{T}$ denote a connected subtree such that $\mathcal{F}_0$ is non-empty. We fix an ideal $\mathfrak{a} \subset A$ and write $d = \deg(\mathfrak{a})$.

Let $\mathcal{T}_+ \subset \mathcal{T}$ denote the vertices of positive type.

DEFINITION: Suppose $v, v' \in \mathcal{T}_+$. We say v' *orbits* v if there is an $m \geq 0$ such that $v = \tau^{(m)}(v')$. Let $\mathcal{T}(v)$ denote the subtree of $\mathcal{T}$ spanned by the vertices orbiting v and $\mathcal{F}(v) = \mathcal{T}(v) \cap \mathcal{F}$. We call it the *constellation* centered at v.

If one imagines bodies orbiting a common point, the following lemma motivates the terminology.

LEMMA 3.0.10. *Suppose $n \geq 1$ and $v \in \mathcal{T}_n$. Then the stabilizer in Γ of $\mathcal{T}(v)$ is Γ_v. It acts transitively on $\mathcal{T}(v)_m$ for every $1 \leq m \leq n$.*

PROOF. The first part follows by observing that $\gamma\mathcal{T}(v) = \mathcal{T}(\gamma v)$ for every $\gamma \in \Gamma$. The second part is corollary 2.0.2. □

The first property in the following definition is motivated by corollary 2.0.9.

DEFINITION: We say $\mathcal{F}$ is a *bipartite* tree if it satisfies the following properties:

- `(B1)`: For every $n \geq d$ and $v \in \mathcal{F}_n$, there are precisely two edges from v in $\mathcal{F}$, one of which is $\varepsilon(v)$.
- `(B2)`: For every $1 < n < d$ and $v \in \mathcal{F}_n$, $\mathcal{F}(v)$ is the union of the segments from $\mathcal{F}(v)_1$ to v.
- `(B3)`: For every $v \in \mathcal{F}_1$, there is an edge from $\mathcal{F}_0$ to v.

For $v \in \mathcal{T}_+$, let $\mathcal{L}(v)$ denote the half-line spanned by $\varepsilon(\tau^{(m)}(v))$ for $m \geq 0$. We call it the cusp starting at v.

LEMMA 3.0.11. *Suppose $\mathcal{F}, \mathcal{F}'$ satisfy* `(B1)` *and* `(B2)`*. Then $\mathcal{F}_{\leq 1} = \mathcal{F}'_{\leq 1}$ if and only if $\mathcal{F} = \mathcal{F}'$.*

PROOF. One direction is trivial: if $\mathcal{F} = \mathcal{F}'$, then $\mathcal{F}_{\leq 1} = \mathcal{F}'_{\leq 1}$. Property `(B1)` is equivalent to assuming that $\mathcal{F}$ is the union of $\mathcal{F}_{\leq d-1}$ and the cusps $\mathcal{L}(v)$ starting at each $v \in \mathcal{F}_{d-1}$. Hence $\mathcal{F} = \mathcal{F}'$ if $\mathcal{F}_{\leq d-1} = \mathcal{F}'_{\leq d-1}$. Both properties imply $\mathcal{F}$ is the union of $\mathcal{F}_{\leq 1}$ and the cusps starting at each $v \in \mathcal{F}_1$. In particular, $\mathcal{F}$ may be recovered from $\mathcal{F}_{\leq 1}$, so $\mathcal{F} = \mathcal{F}'$ if $\mathcal{F}_{\leq 1} = \mathcal{F}'_{\leq 1}$. □

Suppose that $v, v' \in \mathcal{T}_1$. If they do not belong to a common constellation, then we say that v, v' do not meet and define the distance $d(v, v') = \infty$. This happens if and only if the segment between them contains a vertex in $\mathcal{T}_0$. Otherwise, we define $d(v, v')$ to be the radius of the smallest constellation $\mathcal{T}(v'')$ containing both vertices, and call $\mu(v, v') = v''$ the meeting point; if $v'' \in \mathcal{T}_m$, then $d(v, v') = m - 1$.

LEMMA 3.0.12. *If $\mathcal{F}$ satisfies property* `(B1)` *and $v, v' \in \mathcal{F}_1$, then either $d(v, v') \leq d - 2$ or $d(v, v') = \infty$.*

PROOF. The segment from v to v' has trivial intersection with each cusp. Corollary 2.0.9 implies it is confined to $\mathcal{F}_{\leq d-1}$. If $m = d(v, v') < \infty$, then their meeting point lies on this segment, hence is in $\mathcal{F}_{\leq d-1}$. Therefore $m \leq d - 2$. □

Let $\sim$ denote the equivalence relation on $\mathcal{T}_1$ such that $v \sim v'$ if $d(v,v') \leq d-2$ and $\overline{\mathcal{T}}$ the quotient tree $\mathcal{T}_{\leq 1}/\sim$. For any $\gamma \in \Gamma$ we have $\mu(\gamma v, \gamma v') = \gamma v'' = \gamma\mu(v,v')$, hence d is Γ-equivariant. This allows us to "restrict" the action of Γ on $\mathcal{T}_{\leq 1}$ to the quotient $\overline{\mathcal{T}}$. Restricting $\sim$ to any $\mathcal{F}$, we let $\overline{\mathcal{F}}$ denote the quotient tree $\mathcal{F}_{\leq 1}/\sim$.

Corollary 3.0.13. *If $\mathcal{F}$ satisfies* `(B1)`*, then $\overline{\mathcal{F}}$ connected.*

Proof. If $\overline{v}, \overline{v}'$ are vertices in $\overline{\mathcal{F}}$, it suffices to construct the segment connecting $\overline{v}$ to $\overline{v}'$. Let v, v' be vertices in $\mathcal{F}_{\leq 1}$ lying over $\overline{v}, \overline{v}'$, and consider the segment s connecting them in $\mathcal{F}$. Lemma 3.0.12 implies it is confined to $\mathcal{F}_{\leq d-1}$. We construct the segment connecting $\overline{v}$ to $\overline{v}'$ by contracting each subgment of s, contained within a constellation $\mathcal{T}(v'')$ centered in $\mathcal{F}_{d-1}$, to the point $\overline{v}''$ in $\overline{\mathcal{F}}_1$ associated to v''. □

Let $\overline{\mathcal{F}} \subset \overline{\mathcal{T}}$ denote a connected subtree such that $\overline{\mathcal{F}}_0$ is non-empty. One may lift it to $\mathcal{F}_{\leq 1} \subset \mathcal{T}_{\leq 1}$ by lifting the edges, then to $\mathcal{F} \subset \mathcal{T}$ by adjoining $\mathcal{L}(v)$ for every $v \in \mathcal{F}_1$. We call $\mathcal{F}$ the *bipartite lift* of $\overline{\mathcal{F}}$. As the following lemma shows, given a bipartite $\mathcal{F}$, the bipartite lift of $\overline{\mathcal{F}}$ is $\mathcal{F}$ again.

Lemma 3.0.14. *Suppose $\mathcal{F}, \mathcal{F}'$ are bipartite. Then $\overline{\mathcal{F}} = \overline{\mathcal{F}}'$ if and only if $\mathcal{F} = \mathcal{F}'$.*

Proof. One direction is trivial: if $\mathcal{F} = \mathcal{F}'$, then $\overline{\mathcal{F}} = \overline{\mathcal{F}}'$. Suppose that $\overline{\mathcal{F}} = \overline{\mathcal{F}}'$. The edges of $\overline{\mathcal{F}}$ correspond bijectively to those of $\mathcal{F}_{\leq 1}$. Property `(B3)` assert that for every $v \in \mathcal{F}_1$ there is an edge from v to $\mathcal{F}_0$, hence $\mathcal{F}_{\leq 1}$ may be recovered from $\overline{\mathcal{F}}$. In particular, $\mathcal{F}_{\leq 1} = \mathcal{F}'_{\leq 1}$, and lemma 3.0.12 implies $\mathcal{F} = \mathcal{F}'$. □

Fix an arithmetic subgroup $\Delta \supseteq \Gamma(\mathfrak{a})$. If $\mathcal{F}$ is a fundamental domain of Δ, then $\overline{\mathcal{F}}$ is a fundamental domain of Δ acting on $\overline{\mathcal{T}}$. However, if $\overline{\mathcal{F}}$ is a fundamental domain of Δ on $\overline{\mathcal{T}}$, then the bipartite lift $\mathcal{F}$ is *not* necessarily a fundamental domain of Δ on $\mathcal{T}$.

Suppose $\overline{e}, \overline{e}'$ are edges $\overline{\mathcal{T}}_0$ to $\overline{\mathcal{T}}_1$. We lift each to $\mathcal{T}_{\leq 1}$ and let v, v' be the endpoints in $\mathcal{T}_1$. We define

$$d(\overline{e}, \overline{e}') := \begin{cases} 0 & \text{if } \overline{e} = \overline{e}' \\ d(v,v')+1 & \text{otherwise} \end{cases}. \tag{3.0.1}$$

If $\overline{v} \in \overline{\mathcal{T}}_1$ and $v \in \mathcal{T}_{d-1}$ is the center of the associated constellation, then one easily verifies that $\Delta_{\overline{v}} = \Delta_v$.

Definition: We say that $\overline{\mathcal{F}} \subset \overline{\mathcal{T}}$ is Δ-*compressed* if it satisfies the following properties:

- `(D1)`: For every $\overline{v} \in \overline{\mathcal{F}}_1$ and every pair of edges $\overline{e}_1, \overline{e}_2$ from $\overline{\mathcal{F}}_0$ to $\overline{v}$,
$$d(\overline{e}_1, \overline{e}_2) = \min\{d(\overline{e}_1, \overline{e}_3) : \overline{e}_3 \in \Delta_{\overline{v}}\, \overline{e}_2\}.$$
- `(D2)`: For every $\overline{v} \in \overline{\mathcal{F}}_1$ and every $\gamma \in \Delta$, $\gamma\overline{v} \in \overline{\mathcal{F}}_1$ if only if $\gamma \in \Delta_{\overline{v}}$.

Suppose $\overline{e}_1, \overline{e}_2$ are edges from $\overline{\mathcal{F}}_0$ to $\overline{v}$. If they are $\Delta_{\overline{v}}$-equivalent, property `(D1)` implies $d(\overline{e}_1, \overline{e}_2) = 0$, which happens if and only if $\overline{e}_1 = \overline{e}_2$. Thus for every $\overline{v} \in \overline{\mathcal{F}}_1$, the edges from $\overline{\mathcal{F}}_0$ to $\overline{v}$ are pairwise Δ-inequivalent. Property (D2) implies the vertices in $\overline{\mathcal{F}}_1$ are pairwise Δ-inequivalent, hence so are the cusps $\mathcal{L}(v) \subset \mathcal{F}$ for every $v \in \mathcal{F}_{d-1}$. Together the properties imply that the edges of $\overline{\mathcal{F}}$, hence the edges of $\mathcal{F}_{\leq 1}$, are pairwise Δ-inequivalent.

Lemma 3.0.15. *Suppose $\overline{\mathcal{F}} \subset \overline{\mathcal{T}}$ is connected and $\overline{\mathcal{F}}_0$ is non-empty. The edges of the bipartite lift $\mathcal{F}$ are pairwise Δ-inequivalent if and only if $\overline{\mathcal{F}}$ is Δ-compressed.*

Proof. From the remarks preceding the lemma we see that properties (D1) and (D2) are necessary for the edges of $\mathcal{F}$ to be pairwise Δ-inequivalent, hence it suffices to show that they are sufficient.

Suppose $\overline{\mathcal{F}}$ is Δ-compressed and e_1, e_2 are distinct Δ-equivalent edges of $\mathcal{F}$. They must lie in $\mathcal{F}(v)$ for some $v \in \mathcal{F}_{d-1}$ because the edges of $\mathcal{F}_{\leq 1}$ are pairwise Δ-inequivalent. By property (B2) there are (distinct) vertices $v', v'' \in \mathcal{F}(v)_1$ such that e_1, e_2 lie on the segments in $\mathcal{F}(v)$ connecting v', v'' to their meeting point $\mu(v', v'')$. If e_1', e_2' are the edges of the segments ending at $\mu(v', v'')$ and $e_1 = \gamma e_2$ with $\gamma \in \Delta_v$, then $e_1' = \gamma e_2'$, which implies $d(v', \gamma v'') < d(v', v'')$.

By property (B3) there exist distinct edges e_1'', e_2'' from $\mathcal{F}_0$ to v', v''. They contract to distinct edges $\overline{e}_1{}'', \overline{e}_2{}''$ from $\overline{\mathcal{F}}_0$ to the point $\overline{v} \in \overline{\mathcal{F}}_1$ corresponding to v. In particular,

$$d(\overline{e}_1,'', \gamma\overline{e}_2{}'') - 1 = d(v', \gamma v'') < d(v', v'') = d(\overline{e}_1{}'', \overline{e}_2{}'') - 1,$$

which contradicts (D1). The first equality results from (3.0.1), for the edges of $\overline{\mathcal{F}}_{\leq 1}$ are Δ-inequivalent, hence $\overline{e}_1{}'' \neq \gamma\overline{e}_2{}''$. The second follows similarly. □

We now have most of the pieces for the following theorem.

Theorem 3.0.16. *Let $\overline{\mathcal{F}}$ be a connected fundamental domain of Δ acting on $\overline{\mathcal{T}}$. The bipartite lift $\mathcal{F}$ is a connected fundamental domain of Δ acting on $\mathcal{T}$ if and only if $\overline{\mathcal{F}}$ is Δ-compressed.*

Proof. As we observed before, one direction is trivial. It remains to show that the bipartite lift $\mathcal{F}$ of a fundamental domain $\overline{\mathcal{F}}$ of Δ on $\overline{\mathcal{T}}$ is one of Δ on $\mathcal{T}$. By lemma 3.0.15 the edges of $\mathcal{F}$ are pairwise Δ-inequivalent, hence $\mathcal{F}$ is a fundamental domain of Δ on $\mathcal{T}$, unless there is an edge of $\mathcal{T}$ which is Γ-inequivalent to every edge of $\mathcal{F}$.

Suppose e is an edge from $\mathcal{T}_n$ to $\mathcal{T}_{n+1}$. If $n = 0$, the contraction $\overline{e}$ is Δ-equivalent to an edge of $\overline{\mathcal{F}}$, hence e is Δ-equivalent to an edge of $\mathcal{F}$. If $n \geq 1$, we may choose a vertex $v \in \mathcal{T}_1$ such that $e = \varepsilon(\tau^{(n-1)}(v))$ and an edge e' from $\mathcal{T}_0$ to v. By the above, $\gamma e'$ is an edge of $\mathcal{F}$ for some $\gamma \in \Delta$, hence γv is in $\mathcal{F}_1$. Because $\mathcal{F}$ is connected and satisfies (B1) and (B2), the edge $\gamma\varepsilon(\tau^{(m)}(v))$ is in $\mathcal{F}$ for every $m \geq 0$. In particular, γe is in $\mathcal{F}$. □

4. Constructing Fundamental Domains in $\overline{\mathcal{T}}$

Let $\overline{\mathcal{T}}$ be the contracted Bruhat-Tits tree associated to $\Gamma(\mathfrak{a})$ as in section 3 and $\Delta \supseteq \Gamma(\mathfrak{a})$ an arithmetic subgroup. In this section we construct a fundamental domain $\overline{\mathcal{F}}$ of Δ acting on $\overline{\mathcal{T}}$ and show that it is Δ-compressed. If one applies theorem 3.0.16, then the bipartite lift $\mathcal{F}$ will be a connected fundamental domain of Δ acting on $\mathcal{T}$.

Let e_0 the "canonical" edge of $\mathcal{T}$ from v_0 to v_1 (cf. section 2). We denote the contracted edge by $\overline{e}_0$ and the contracted vertices by $\overline{v}_i$. Let $B \subset \Gamma_0$ denote the Borel subgroup of upper triangular matrices. We recall the following facts without proof (cf. lemmas 2.0.5 and 3.0.10).

Lemma 4.0.17. $\Gamma_{\overline{v}_0} = \Gamma_0$, $\Gamma_{\overline{v}_1} = \Gamma_{d-1}$, *and* $\Gamma_{\overline{e}_0} = \Gamma_{\overline{v}_0} \cap \Gamma_{\overline{v}_1} = B$.

We refer to vertices in $\overline{\mathcal{T}}_0$ as *nodes* and those in $\overline{\mathcal{T}}_1$ as *cusps*. We call an edge $\overline{e}$ from a node to a cusp a *positive* edge, and we associate to it the coset γB where $\gamma \in \Gamma$ satisfies $\gamma\overline{e}_0 = \overline{e}$. We obtain a bijection with the coset space Γ/B.

Corollary 4.0.18. $\{$ *positive edges of* $\overline{\mathcal{T}}$ $\} \xrightarrow{1-1} \{$ *cosets of* Γ/B $\}$.

The cosets $\Gamma_{\overline{v}_0}/B$ correspond bijectively to the edges of $\overline{\mathcal{T}}$ from $\overline{v}_0$ and those of $\Gamma_{\overline{v}_1}/B$ to the edges of $\overline{\mathcal{T}}$ to $\overline{v}_1$. We choose representatives of the cosets as follows.

Viewing the coset space $\Gamma_{\overline{v}_0}/B$ as $\mathbb{P}^1(k)$, we index the cosets by z, and let $\{\omega_z\}$ be the coset representatives

$$\left\{ \omega_a = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} : a \in k \right\} \cup \left\{ \omega_\infty = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\}.$$

B is a normal subgroup of $\Gamma_{\overline{v}_1}$, and the coset space $\Gamma_{\overline{v}_1}/B$ is a group isomorphic to $R := k[t]/t^{d-1}$. We index the cosets by $b \in R$, so $\deg(b) \leq d-2$, and let $\{\gamma_b\}$ be the coset representatives

$$\left\{ \gamma_b = \begin{pmatrix} 1 & t \cdot b \\ 0 & 1 \end{pmatrix} \right\}.$$

Let $S_0 = \{\omega_z\} - \{\omega_0\}$, $S_1 = \{\gamma_b\} - \{\gamma_0\}$ and S be the disjoint union $S_0 \cup S_1$. We refer to elements of S as *letters*, elements of S_0 as *nodal* letters and elements of S_1 as *cuspidal* letters.

Definition: An *S-word* is a finite product $w = \lambda_1 \cdots \lambda_n$ of letters. When viewed as an element of Γ, the empty word corresponds to the identity element. We say that an S-word is *reduced* if every pair of successive letters λ_i, λ_{i+1} has both a nodal and a cuspidal letter. Moreover, we say it is *nodal* (resp. *cuspidal*) if the last letter is nodal (resp. cuspidal); the empty word is both nodal and cuspidal.

To every S-word w we associate the edge $\overline{e}_w = w\,\overline{e}_0$ in $\overline{\mathcal{T}}$, associating $\overline{e}_0$ to the empty word.

Lemma 4.0.19. *Suppose w is a reduced S-word.*

(1) *If w is cuspidal and $\overline{v}$ is the node of $\overline{e}_w$, then the nodal letters $\lambda \in S_0$ correspond bijectively to the other edges of $\overline{\mathcal{T}}$ from $\overline{v}$ via $\lambda \mapsto \overline{e}_{w\cdot\lambda}$.*
(2) *If w is nodal and $\overline{v}$ is the cusp of $\overline{e}_w$, then the cuspidal letters $\lambda \in S_1$ correspond bijectively to the other edges of $\overline{\mathcal{T}}$ to $\overline{v}$ via $\lambda \mapsto \overline{e}_{w\cdot\lambda}$.*

Proof. We will prove the first statement, leaving the analogous proof of the second statement to the reader. Let w be a cuspidal (reduced) S-word and $\overline{v}$ the node of $\overline{e}_w$. Suppose $\lambda \in S_0$ and $w' = w \cdot \lambda$. Then $w\lambda w^{-1}$ is an element of $\Gamma_{\overline{v}}$, hence

$$\overline{e}_{w'} = (w\lambda)\,\overline{e}_0 = (w\lambda w^{-1}w)\,\overline{e}_0 = (w\lambda w^{-1})\,\overline{e}_w$$

is an edge whose node is also $\overline{v}$. The coset $w'B$ is distinct from wB because $w^{-1} \cdot w' = \lambda \notin B$, hence $\overline{e}_w$ and $\overline{e}_{w'}$ are distinct edges by corollary 4.0.18. If $\lambda, \lambda' \in S_0$ are distinct, then $(w\cdot\lambda)^{-1}(w\cdot\lambda') = \lambda^{-1}\lambda' \notin B$, hence $\overline{e}_{w\cdot\lambda}$ and $\overline{e}_{w\cdot\lambda'}$ are distinct edges of $\overline{\mathcal{T}}$ to $\overline{v}$. □

We say that two edges of $\overline{\mathcal{T}}$ are adjacent if they are distinct and have a vertex in common.

Corollary 4.0.20. $\{$ *reduced S-words* $\} \xrightarrow{1-1} \{$ *positive edges of $\overline{\mathcal{T}}$* $\}$.

Proof. Suppose $\overline{e}$ is a positive edge of $\overline{\mathcal{T}}$. Let $\overline{e}_0, \ldots, \overline{e}_n$ be the unique sequence of adjacent edges in $\overline{\mathcal{T}}$ from $\overline{e}_0$ to $\overline{e}_n = \overline{e}$. By lemma 4.0.19 and induction on i, there is a unique reduced S-word $w = \lambda_1 \cdots \lambda_n$ such that $\overline{e}_i = \overline{e}_{\lambda_1\cdots\lambda_i}$ for $0 \le i \le n$. □

We denote the set of reduced S-words by Σ. Combining corollaries 4.0.18 and 4.0.20 we see that Σ is a set of coset representatives of Γ/B.

Lemma 4.0.21. *Suppose $w, w' \in \Sigma$ are distinct. If $\overline{e}_w, \overline{e}_{w'}$ are not adjacent edges or their common vertex is a node, then $d(\overline{e}_w, \overline{e}_{w'}) = \infty$. Otherwise, $w^{-1}w'B = \gamma_b B$ for a unique $\gamma_b \in S_1$ and $d(\overline{e}_w, \overline{e}_{w'}) = \deg(b) + 1$.*

Proof. The first part follows from the definition of $d(\overline{e}_w, \overline{e}_{w'})$ (cf. section 3). Suppose have the cusp $\overline{v}$ in common. We may assume either w is nodal or both w, w' are cuspidal.

In the first case, $w' = w\gamma_b$ for a unique $\gamma_b \in S_1$, by part 2 of lemma 4.0.19. Because $d(\cdot,\cdot)$ is Γ-equivariant we have

$$d(\overline{e}_w, \overline{e}_{w'}) = d(w\overline{e}_0, w'\overline{e}_0) = d(w\overline{e}_0, w\gamma_b\overline{e}_0) = d(\overline{e}_0, \gamma_b\overline{e}_0).$$

Therefore it suffices to prove $d(\overline{e}_0, \gamma_b\overline{e}_0) = \deg(b) + 1$ for every $\gamma_b \in S_1$.

In the second case $w = w''\gamma_{b'}$ and $w' = w''\gamma_{b''}$ for a unique nodal $w'' \in \Sigma$ and distinct $\gamma_{b'}, \gamma_{b''} \in S_1$, again by part 2 of lemma 4.0.19. Writing $b = b'' - b'$ we have

$$d(\overline{e}_w, \overline{e}_{w'}) = d(w''\gamma_{b'}\overline{e}_0, w''\gamma_{b''}\overline{e}_0) = d(\overline{e}_0, \gamma_b\overline{e}_0).$$

Again it suffices to prove that $d(\overline{e}_0, \gamma_b\overline{e}_0) = \deg(b) + 1$.

Let $\overline{\mathcal{F}}$ be the union of the edges $\overline{e}_0, \gamma_b\overline{e}_0$ and $\mathcal{F}$ the bipartite lift. We denote the lifts of the cusps of $\overline{e}_0, \gamma_b\overline{e}_0$ by $v, \gamma_b v$. From the definition of γ_b we see that it is an element of $\Gamma_{\deg(b)+1} - \Gamma_{\deg(b)}$. Moreover, their meeting point $m(v, \gamma_b v)$ is the only vertex fixed by γ_b on the segments from $v, \gamma_b v$ to v'. By lemma 2.0.1 we must have $m(v, \gamma_b v) \in \mathcal{T}_{\deg(b)+1}$, so $d(v, \gamma_b v) = \deg(b)$. Finally, the identity $d(\overline{e}_0, \gamma_b \overline{e}_0) = d(v, \gamma_b v) + 1$ implies the lemma. □

Recall that $R = k[t]/t^{d-1}$. For any element $f \in R$ and $0 \leq n \leq \deg(f)$, let $f_n \in k$ denote the nth coefficient of f, so that $f(t) = \sum_n f_n \cdot t^n$ in R. We choose a bijection $\sigma : k^\times \to \{1, \dots, q-1\}$ and impose the lexical ordering on R as follows.

DEFINITION: If $f, g \in R$ are distinct elements, then we define $f < g$ if either of the following conditions hold:

(O1): $\deg(f) < \deg(g)$;
(O2): $\deg(f) = \deg(g)$ and $\sigma(f_{\deg(f-g)}) < \sigma(g_{\deg(f-g)})$.

We consider the induced order on the cuspidal letters: $\gamma_b < \gamma_{b'}$ if $b < b'$.

We call a positive edge *nodal* (resp. *cuspidal*) if the corresponding reduced S-word w is nodal (resp. cuspidal); $\overline{e}_0$ is the unique edge which is both cuspidal and nodal. Every cusp $\overline{v}$ of $\overline{\mathcal{T}}$ belongs to a unique nodal edge $\overline{e}_w$, for lemma 4.0.19 implies that the remaining edges to $\overline{v}$ correspond bijectively to cuspidal letters via $\gamma_b \mapsto \overline{e}_{w \cdot \gamma_b}$. We consider the induced order on the positive edges of $\overline{\mathcal{T}}$ to $\overline{v}$: $\overline{e}_w < \overline{e}_{w \cdot \gamma_b} < \overline{e}_{w \cdot \gamma_{b'}}$ if $\gamma_0 < \gamma_b < \gamma_{b'}$.

Let $\overline{\mathcal{F}} \subset \overline{\mathcal{T}}$ be connected and assume $\overline{\mathcal{F}}_0$ is non-empty. For any cusp $\overline{v}$ let $\overline{\mathcal{F}}(\overline{v})$ denote the union of the edges containing $\overline{v}$.

LEMMA 4.0.22. *If $\overline{v}$ is a cusp of $\overline{\mathcal{T}}$ and $\overline{e}_1 \leq \overline{e}_2 \leq \overline{e}_3$ are positive edges of $\overline{\mathcal{T}}(\overline{v})$, then*

$$d(\overline{e}_1, \overline{e}_2) \leq d(\overline{e}_1, \overline{e}_3).$$

PROOF. The lemma follows immediately when $\overline{e}_1 = \overline{e}_2$ or $\overline{e}_2 = \overline{e}_3$, hence we assume $\overline{e}_1 < \overline{e}_2 < \overline{e}_3$. It suffices to prove the following: if $\overline{e}_1 < \overline{e}_2, \overline{e}_3$ and $d(\overline{e}_1, \overline{e}_2) > d(\overline{e}_1, \overline{e}_3)$, then $\overline{e}_2 > \overline{e}_3$.

Let $\overline{e}_w$ be the unique nodal edge adjacent to every $\overline{e}_i$ and $b_1 < b_2, b_3$ the unique elements such that $\overline{e}_i = \overline{e}_{w \cdot \gamma_{b_i}}$; $b_1 = 0$ if and only if $\overline{e}_1 = \overline{e}_{w \cdot \gamma_0} = \overline{e}_w$. Let $d_i = \deg(b_i)$ and $d_{i,j} = \deg(b_i - b_j)$. We observe that

$$d_{3,1} = d(\overline{e}_1, \overline{e}_3) - 1 < d(\overline{e}_1, \overline{e}_2) - 1 = d_{2,1},$$

by assumption. Also, by assumption, $\overline{e}_1 < \overline{e}_2, \overline{e}_3$, so lemma 4.0.21 implies $d_1 \leq d_2, d_3$. If $d_1 < d_3$, then

$$d_3 = d_{3,1} < d_{2,1} \leq d_2,$$

so $\overline{e}_2 > \overline{e}_3$. If $d_1 = d_3 < d_2$, then $\overline{e}_2 > \overline{e}_3$. Hence we may assume $d_1 = d_2 = d_3$.

Because $d_{3,1} < d_{2,1}$, b_3 and b_1 have more leading coefficients in common than b_2 and b_1 do. That is, b_3 is closer than b_2 to b_1, in the lexical order. Finally, b_1 precedes both b_2 and b_3, hence b_3 must precede b_2. That is, $\overline{e}_2 > \overline{e}_3$. □

The following is the main theorem of the section.

THEOREM 4.0.23. *Assume $\overline{\mathcal{F}}$ satisfies* (D2). *Suppose for every cusp $\overline{v}$ that $\overline{\mathcal{F}}(\overline{v})$ satisfies the following properties:*

(1) *the positive edges of $\overline{\mathcal{F}}(\overline{v})$ are Δ-inequivalent;*
(2) *for every positive edge $\overline{e}$ of $\overline{\mathcal{F}}(\overline{v})$ we have $\overline{e} = \min\{\gamma\overline{e} : \gamma \in \Delta_{\overline{v}}\}$.*

Then $\overline{\mathcal{F}}$ is Δ-compressed.

PROOF. We must show that $\overline{\mathcal{F}}$ satisfies (D1): for every cusp $\overline{v}$ of $\overline{\mathcal{F}}$ and every pair of positive edges $\overline{e}_1, \overline{e}_2$ in $\overline{\mathcal{F}}(\overline{v})$,

$$d(\overline{e}_1, \overline{e}_2) = \min\{d(\overline{e}_1, \overline{e}_3) : \overline{e}_3 \in \Delta_{\overline{v}}\,\overline{e}_2\}. \tag{4.0.2}$$

Suppose $\overline{e}_1 < \overline{e}_2$ are positive edges of $\overline{\mathcal{F}}(\overline{v})$. By assumption $\overline{e}_2 \leq \gamma\overline{e}_2$ for every $\gamma \in \Delta_{\overline{v}}$, hence applying lemma 4.0.22 with $\overline{e}_3 = \gamma\overline{e}_2$ we obtain (4.0.2) as desired.

For $\overline{e}_2 < \overline{e}_1$ we observe that (4.0.2) is symmetric because $d(\cdot,\cdot)$ is Γ-equivariant and symmetric:

$$\min\{\, d(\overline{e}_1, \gamma\overline{e}_2) : \gamma \in \Delta_{\overline{v}}\,\} = \min\{\, d(\overline{e}_2, \gamma^{-1}\overline{e}_1) : \gamma^{-1} \in \Delta_{\overline{v}}\,\}.$$

Hence we may apply the above argument swapping $\overline{e}_1$ and $\overline{e}_2$. □

It is now quite simple to describe an algorithm for constructing $\overline{\mathcal{F}} = \overline{\mathcal{F}}(\Delta)$ inductively. We choose a node $\overline{v}$ of $\overline{\mathcal{T}}$, say $\overline{v} = \overline{v}_0$, and let $\overline{\mathcal{F}}_{(0)}$ denote the tree whose single vertex is $\overline{v}$. Without loss of generality, we may assume $\overline{v}$ is in $\overline{\mathcal{F}}$. Because $\overline{\mathcal{F}}$ has a finite number of edges, the theorem implies it suffices to construct a strictly increasing sequence of Δ-compressed trees $\overline{\mathcal{F}}_{(0)} \subset \cdots \subset \overline{\mathcal{F}}_{(m)} = \overline{\mathcal{F}}$.

DEFINITION: Suppose $\overline{v}\,'$ is a cusp in $\overline{\mathcal{T}}$. Let $\overline{\mathcal{M}}(\overline{v}\,')$ denote the maximal subtree of $\overline{\mathcal{T}}(\overline{v}\,')$ satisfying property 2 of the theorem.

The idea is to choose a sequence of Δ-inequivalent cusps $\overline{v}\,'_1, \ldots, \overline{v}\,'_m$ in $\overline{\mathcal{T}}$ such that $\overline{\mathcal{M}}(\overline{v}\,'_{n+1})$ is is adjacent to $\overline{\mathcal{F}}_{(n)}$ and to define

$$\overline{\mathcal{F}}_{(n+1)} := \overline{\mathcal{F}}_{(n)} \cup \overline{\mathcal{M}}(\overline{v}\,'_{n+1}).$$

Such a sequence always exists: if $\overline{e}$ is an edge from $\overline{\mathcal{F}}_{(n)}$ to a cusp $\overline{v}\,'_{n+1}$ of $\overline{\mathcal{T}} - \Delta\overline{\mathcal{F}}_{(n)}$, then it is the unique nodal edge in $\overline{\mathcal{T}}(\overline{v}\,'_{n+1})$, so lies in $\overline{\mathcal{M}}(\overline{v}\,'_{n+1})$. Further, the sequence is finite, because the number of cusps m is known to be finite. One uses the following lemma to show inductively that $\overline{\mathcal{F}}_{(n)}$ is Δ-compressed for all n.

LEMMA 4.0.24. *Suppose $\overline{\mathcal{F}}, \overline{\mathcal{M}}$ satisfy the hypothesis of theorem 4.0.23. If their cusps are pairwise Δ-inequivalent, then $\overline{\mathcal{F}} \cup \overline{\mathcal{M}}$ satisfies the hypothesis of the theorem.*

PROOF. The edges of $\overline{\mathcal{F}}, \overline{\mathcal{M}}$ are pairwise Δ-inequivalent, because their cusps are pairwise Δ-inequivalent. Let $\overline{\mathcal{G}}$ denote $\overline{\mathcal{F}} \cup \overline{\mathcal{M}}$. By assumption, it satisfies (D2). Any cusp $\overline{v}$ of $\overline{\mathcal{G}}$ is either a cusp of $\overline{\mathcal{F}}$ or a cusp of $\overline{\mathcal{M}}$. In the first case $\overline{\mathcal{G}}(\overline{v}) = \overline{\mathcal{F}}(\overline{v})$ and in the second $\overline{\mathcal{G}}(\overline{v}) = \overline{\mathcal{M}}(\overline{v})$. In either case $\overline{\mathcal{G}}(\overline{v})$ satisfies the properties in theorem 4.0.23, hence $\overline{\mathcal{G}}$ is Δ-compressed. □

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